1. Find y if

$$\frac{dy}{dx} = x^2 - e^x \qquad \text{and} \qquad y(0) = 4.$$

2. Find $F(t)$ if

$$F'(t) = \frac{4}{1+t^2} \qquad \text{and} \qquad F(1) = 0.$$

3. Find z if

$$\frac{dz}{dq} = \cos(2q) \qquad \text{and} \qquad z(0) = -12.$$

4. A factory produces steel rods. Unfortunately, the factory is dealing with some quality control issues. Some batches of rods come out heavier than others. Factory workers must test one rod from each batch to determine if the batch passes the weight requirement. Recall that for objects with uniform density, mass is given by $m = \rho \cdot \ell$ where ρ is the density of the object and ℓ is the length of the object.

   (a) The ideal rod is 3 m long and has uniform density of 8,000 kg/m. What is the mass of the ideal rod?

   (b) A test rod picked from the first batch has densities 7,500 kg/m for the first meter, 9,500 kg/m for the second meter, and 8,200 kg/m for the third meter. The test rod must be within 1000 kg of the ideal rod to pass inspection. Does the first batch pass inspection? Explain.

   (c) A test rod is picked from the second batch has density given by $\rho(x) = 3500\,(2 + \sin(\pi x))$, where x is measured in m and ρ is measured in kg/m. Set up, but do not evaluate, a definite integral to measure the mass of this test rod.

(d) Evaluate the integral in part (c). Hint: An antiderivative of $\sin(\pi x)$ is $-\dfrac{\cos(\pi x)}{\pi}$.

(e) The test rod for the second batch must be within 1000 kg of the ideal rod to pass inspection. Use your answer to part (d) to determine if the second batch passes inspection. Explain.

5. Find the following indefinite integrals.

(a) $\displaystyle\int \left(2\cos(3x) + e^{7x}\right)\, dx$

(b) $\displaystyle\int \left(e^{2x} + 4^x + x^{3/5}\right)\, dx$