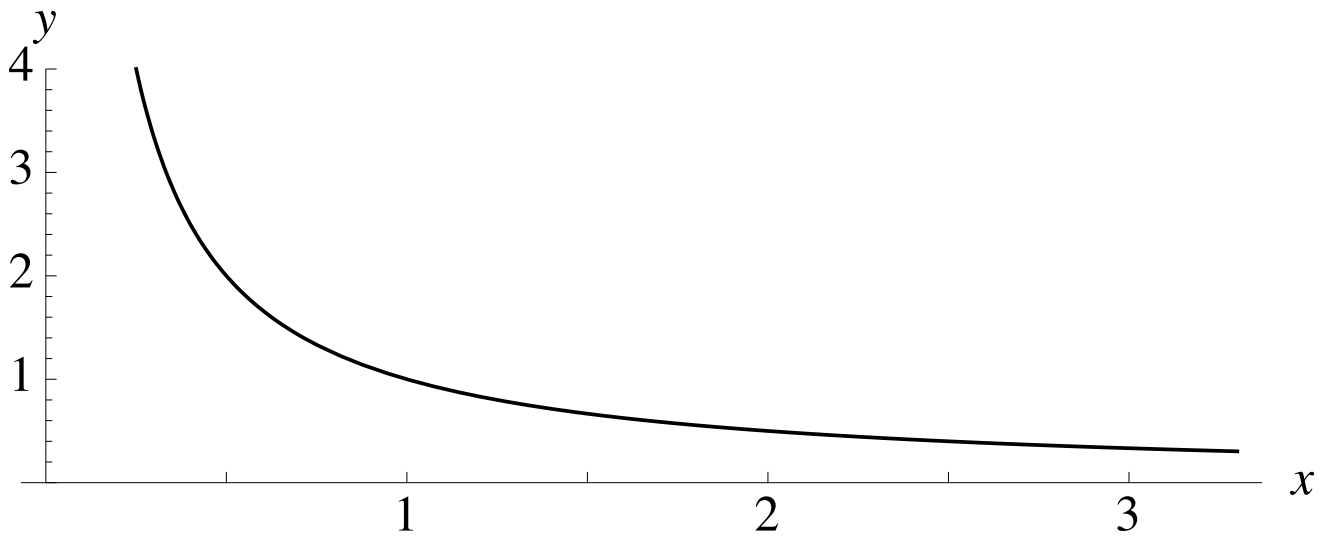
1. On the axes below is the graph of $f(x) = \dfrac{1}{x}$.



(a) Draw, directly on top of this graph, rectangles representing a *left* endpoint Riemann sum approximation, with $n = 4$, to $\displaystyle\int_1^3 f(x)\,dx$.

(b) Approximate $\displaystyle\int_1^3 f(x)\,dx$, using the left endpoint approximation (with $n = 4$) that you represented graphically in the previous part of this problem. Express your answer in decimal form, with at least four digits to the right of the decimal point.

(c) Repeat part (b) using right endpoints. (You don't have to draw the right endpoint rectangles; just do the approximation.)

(d) Find $\int_1^3 f(x)\,dx$ exactly, using the Fundamental Theorem of Calculus. Express your answer in terms of ln(3).

(e) Which of the following numbers is larger:

$$\frac{1}{2}\left(\frac{1}{1}+\frac{1}{1.5}+\frac{1}{2}+\frac{1}{2.5}+\frac{1}{3}+\frac{1}{3.5}+\cdots+\frac{1}{99}+\frac{1}{99.5}\right) \quad \text{or} \quad \ln(100)?$$

Please explain carefully. Hint: imagine that your picture from part (a) extends out to $x = 100$, instead of just $x = 3$.

2. Evaluate each of the following definite integrals. Please express your answers as decimals, with at least four digits to the right of the decimal point.

(a) $\displaystyle\int_{-2}^{-1}\left(x+\frac{1}{x}\right)\,dx$

(b) $\displaystyle\int_{0}^{2} 3^t\,dt$

(c) $\displaystyle\int_{0}^{\pi/2}\cos(3x)\,dx$

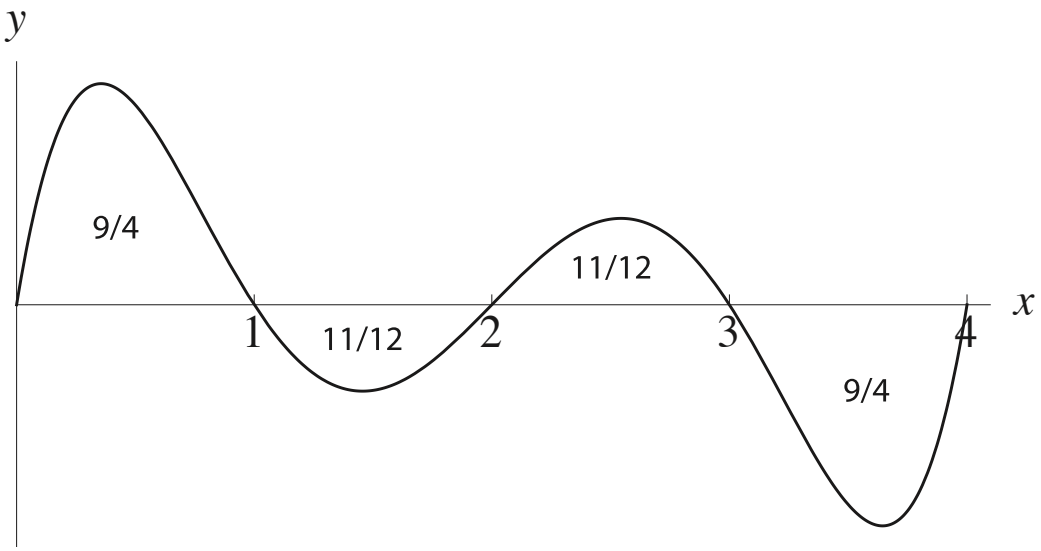
3. Evaluate each of the following indefinite integrals.

(a) $\displaystyle\int\left(\frac{2}{x}+\frac{2}{x^2}+\frac{2}{1+x^2}\right)dx$

(b) $\displaystyle\int\left(5z^3-\frac{7}{z^3}+\sqrt[3]{z}-\frac{1}{\sqrt[3]{z}}\right)dz$

(c) $\displaystyle\int\left(x^2+2^x+2^2\right)dx$

4. Given that the indicated regions have the specified areas:



evaluate the following definite integrals. Please express each answer as a single fraction, or as a decimal with at least four digits to the right of the decimal point.

(a) $\displaystyle\int_0^1 f(x)\,dx$

(b) $\displaystyle\int_0^2 f(x)\,dx$

(c) $\displaystyle\int_1^4 f(x)\,dx$

(d) $\displaystyle\int_4^2 f(x)\,dx$

5. Solve the initial value problem

$$\frac{dy}{dx} = 4x^3 + \frac{2}{x^2}, \qquad y(1) = 6.$$

6. (a) The CU student body consumes coffee at a rate of 5 gallons per hour. Calculate the total coffee consumed by this group in a 24 hour period. Please express your answer as a whole number, and don't forget to include units.

(b) Now suppose these students actually consume 8 gallons per hour between 6 am and 11 am, then consume 5 gallons per hour between 11 am and 4 pm, and finally they consume 3 gallons per hour between 4 pm and 10 pm. Calculate the total coffee consumed by this group between 6 am and 11 pm. Again, please express your answer as a whole number, and don't forget to include units.

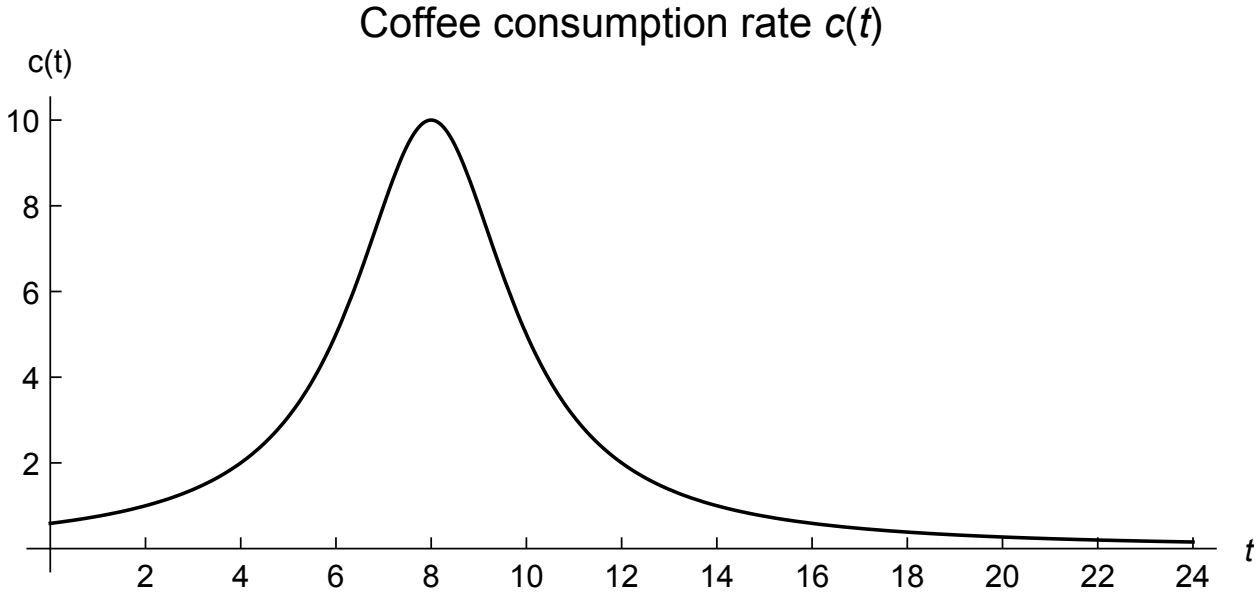
(c) Suppose students consume coffee at a rate $c(t)$ given

$$c(t) = \frac{10}{1 + (.5t - 4)^2}$$

gallons per hour, where t is the number of hours since midnight.

Set up but DO NOT EVALUATE an integral to represent the total coffee consumed between 3 am and 6 pm. What are the units of the quantity represented by this integral?

(d) Below is the graph of the function $c(t)$ from part (c) above.



i. On the above graph, shade in the area that represents your answer to part (c) above.

ii. At what point in time, between 3 am and 6 pm, did CU students consume coffee most rapidly? By referring to the graph, please circle the correct answer, and explain.

3 am 11 am 8 am 6 pm 12 pm (noon)

iii. Using the graph above, determine whether the following is true or false: Between 3 am and 6 pm, CU students consumed more than 150 gallons of coffee. (Assume, again, that coffee is consumed at the rate $c(t)$ given by the above graph.) Please circle the correct answer, and explain.

TRUE FALSE