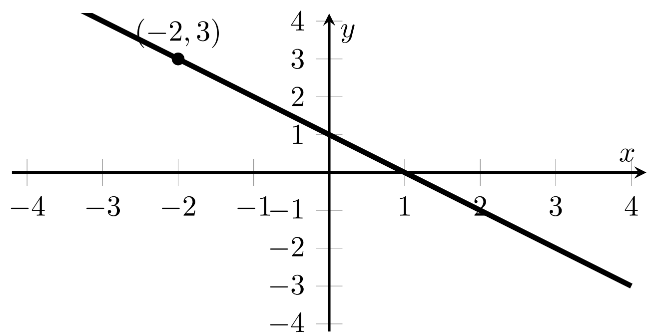
1. Below is the graph of a function $f(x)$. Use geometry to compute the following definite integrals.

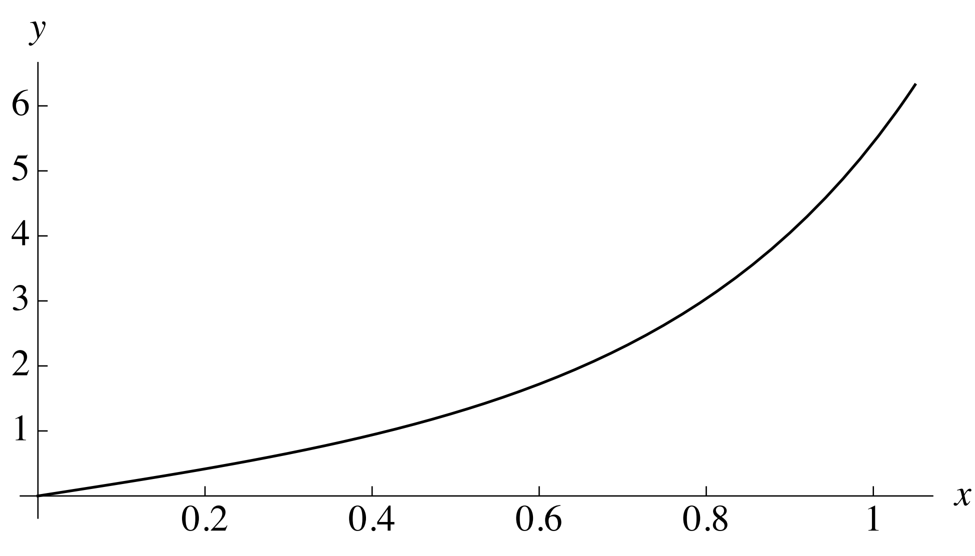


(a) $\int_{-3}^{3} 5f(x)dx$

(b) $\int_{0}^{2} f(x)dx$

(c) $\int_{0}^{2} |f(x)|dx$

2. A graph of the function $f(x) = 2xe^{x^2}$ is shown below.



(a) On top of the above graph, draw the rectangles corresponding to a right endpoint Riemann sum approximation, with $n = 5$ rectangles, to the area

$$\int_0^1 f(x)\,dx.$$

(b) Use the rectangles above to provide a Riemann sum approximation to $\int_0^1 f(x)\,dx$. Please supply at least four places to the right of the decimal point.

$$\int_0^1 f(x)\,dx \approx$$

(c) Is your approximation from part (b) above an overestimate or an underestimate? Please explain geometrically (by referring to the above picture), *without* actually computing the exact area (yet).

(d) Given that $F(x) = e^{x^2}$ is an antiderivative of $f(x) = 2xe^{x^2}$, calculate the actual area $\int_0^1 f(x)\,dx$. Please provide an answer in terms of e, *and* supply a decimal answer with at least four places to the right of the decimal point.

$$\int_0^1 f(x)\,dx =$$

3. Last winter was cold, so Dr. Who bought a multi-watt space heater.

(a) During the first month, Dr. Who ran the space heater day and night, at 2000 watts. How much energy, in watt-hours (wh), was Dr. Who using per day (from midnight to midnight), during the first month?

(b) When Dr. Who saw his electric bill after the first month, he decided that it would be a good idea to use the multi-watt settings. So each day during the next month, Dr. Who ran his space heater at 500 watts from 12am to 8am, turned it off from 8am to 12pm, ran it at 500 watts from 12 pm to 6 pm, and ran it at 2000 watts from 6pm to 12am. How much energy, in watt-hours (wh), was Dr. Who using per day (from midnight to midnight), during the second month?

(c) To maximize energy efficiency, Dr. Who buys a regulator for his heater, so that it generates power at a rate of

$$f(t) = 1000\left(1 + \cos\left(\frac{\pi}{12}t\right)\right)$$

watts, where t is measured in hours (starting at midnight).

(i) Set up, but do *not* evaluate, a definite integral to measure how much energy Dr. Who's heater, with the new regulator, uses over the course of a day (from midnight to midnight).

(ii) Evaluate the integral in part (i), to four decimal places. Hint: An antiderivative of $\cos\left(\frac{\pi}{12}t\right)$ is

$$\frac{12}{\pi}\sin\left(\frac{\pi}{12}t\right).$$

What are the units for your answer?

4. An object is pushed from a point $x = 0$ to $x = 10$, with a varying force of

$$F(x) = 2 + \cos(x)$$

pounds (here, x is measured in feet).

(a) What is the amount of work $W(X)$ done in pushing the object from $x = 0$ to $x = X$ (where X is any number between 0 and 10)? What are the units of $W(X)$? (It may help to recall that, if a *constant* force F is applied over a distance d, then the work W done is given by $W = F \cdot d$.)

(b) At what point or points along the way, from $x = 0$ to $x = 10$, is W changing most rapidly? What *is* the rate of change of $W(X)$ at these points? Hint: $\cos(\theta)$ is largest when $\theta = 0, \pm 2\pi, \pm 4\pi, \ldots$.

5. Compute the following indefinite integrals.

(a) $\int e^{3x}\,dx$

(b) $\int \left(\frac{3}{x} + \frac{1}{x^4} + \sqrt[5]{x}\right) dx$

(c) $\int (\pi \sin(y) + 3)\,dy$

(d) $\int 5 \cdot 4^z\,dz$

6. Compute the following definite integrals. Please leave numbers like e^2, $\ln(3)$, $\sin(6)$, etc. in exact form; that is, you don't need to evaluate these numbers as decimals on your calculator.

(a) $\int_0^2 (5e^x + \pi)\,dx$

(b) $\int_\pi^{2\pi} \left(\frac{1}{x} + \sin(x) + x^{-2/3}\right) dx$

(c) $\int_0^3 2^{y/3}\,dy$

(d) $\int_{-1}^{1} \frac{2}{1+z^2}\,dz$ (Hint: $\tan(-\pi/4) = -1$ and $\tan(\pi/4) = 1$.)

7. Find z if

$$\frac{dz}{dq} = \cos(2q) \qquad \text{and} \qquad z(0) = -12.$$

8. A factory produces steel rods. Unfortunately, the factory is dealing with some quality control issues. Some batches of rods come out heavier than others. Factory workers must test one rod from each batch to determine if the batch passes the mass requirement. Recall that for objects with uniform density, mass is given by $m = \rho \cdot \ell$ where ρ is the density of the object and ℓ is the length of the object.

(a) The ideal rod is 3 m long and has uniform density of 8,000 kg/m. What is the mass of the ideal rod?

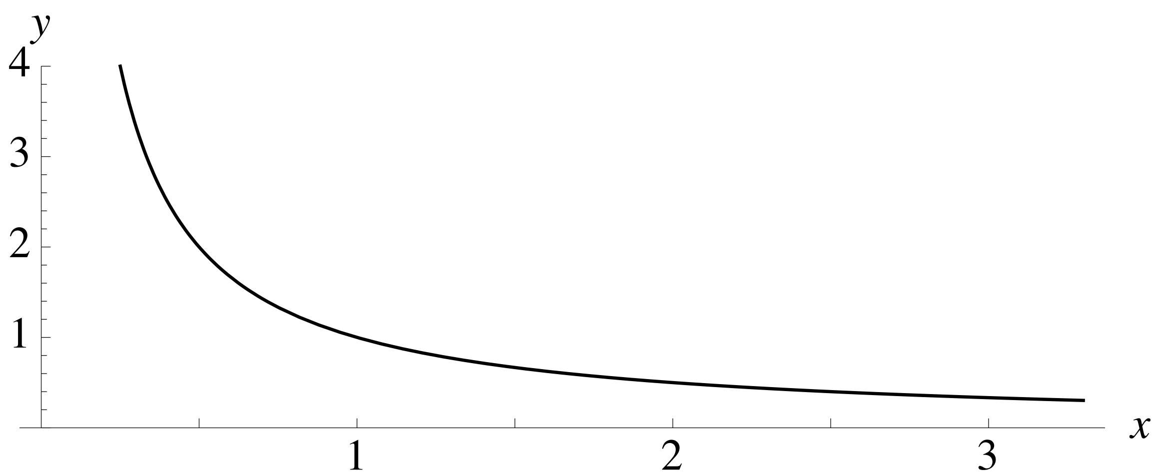
(b) A test rod picked from the first batch has densities 7,500 kg/m for the first meter, 9,500 kg/m for the second meter, and 8,200 kg/m for the third meter. The test rod must be within 1000 kg of the ideal rod to pass inspection. Does the first batch pass inspection? Explain.

(c) A test rod is picked from the second batch has density given by $\rho(x) = 3500\,(2 + \sin(\pi x))$, where x is measured in m and ρ is measured in kg/m. Set up, but do not evaluate, a definite integral to measure the mass of this test rod.

(d) Evaluate the integral in part (c). Hint: An antiderivative of $\sin(\pi x)$ is $-\dfrac{\cos(\pi x)}{\pi}$.

(e) The test rod for the second batch must be within 1000 kg of the ideal rod to pass inspection. Use your answer to part (d) to determine if the second batch passes inspection. Explain.

9. On the axes below is the graph of $f(x) = \dfrac{1}{x}$.



(a) Draw, directly on top of this graph, rectangles representing a *left* endpoint Riemann sum approximation, with $n = 4$, to $\int_1^3 f(x)\,dx$.

(b) Approximate $\int_1^3 f(x)\,dx$, using the left endpoint approximation (with $n = 4$) that you represented graphically in the previous part of this problem. Express your answer in decimal form, with at least four digits to the right of the decimal point.

(c) Repeat part (b) using right endpoints. (You don't have to draw the right endpoint rectangles; just do the approximation.)

(d) Find $\int_1^3 f(x)\,dx$ exactly, using the Fundamental Theorem of Calculus. Express your answer in terms of ln(3).

(e) Which of the following numbers is larger:

$$\frac{1}{2}\left(\frac{1}{1}+\frac{1}{1.5}+\frac{1}{2}+\frac{1}{2.5}+\frac{1}{3}+\frac{1}{3.5}+\cdots+\frac{1}{99}+\frac{1}{99.5}\right) \quad \text{or} \quad \ln(100)?$$

Please explain carefully. Hint: imagine that your picture from part (a) extends out to $x = 100$, instead of just $x = 3$.

10. Evaluate each of the following definite integrals. Please express your answers as decimals, with at least four digits to the right of the decimal point.

(a) $\int_{-2}^{-1} \left(x + \frac{1}{x}\right) dx$

(b) $\int_0^2 3^t\,dt$

(c) $\int_0^{\pi/2} \cos(3x)\,dx$

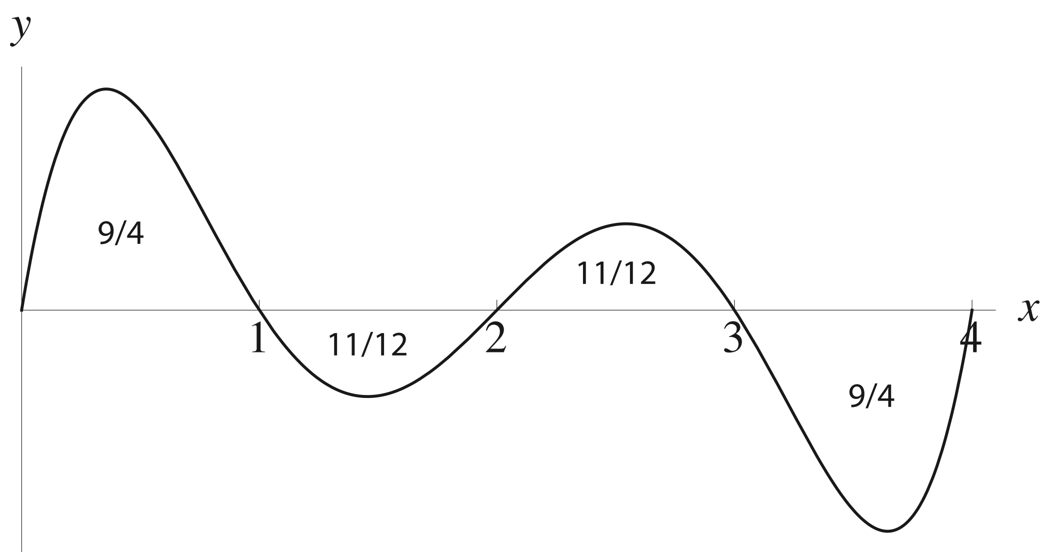
11. Evaluate each of the following indefinite integrals.

(a) $\int \left(\frac{2}{x} + \frac{2}{x^2} + \frac{2}{1+x^2} \right) dx$

(b) $\int \left(5z^3 - \frac{7}{z^3} + \sqrt[3]{z} - \frac{1}{\sqrt[3]{z}} \right) dz$

(c) $\int \left(x^2 + 2^x + 2^2 \right) dx$

12. Given that the indicated regions have the specified areas:



evaluate the following definite integrals. Please express each answer as a single fraction, or as a decimal with at least four digits to the right of the decimal point.

(a) $\int_0^1 f(x)\,dx$ _____

(b) $\int_0^2 f(x)\,dx$ _____

(c) $\int_1^4 f(x)\,dx$_______

(d) $\int_4^2 f(x)\,dx$_____

13. Solve the initial value problem

$$\frac{dy}{dx} = 4x^3 + \frac{2}{x^2}, \qquad y(1) = 6.$$

14. (a) The CU student body consumes coffee at a rate of 5 gallons per hour. Calculate the total coffee consumed by this group in a 24 hour period. Please express your answer as a whole number, and don't forget to include units.

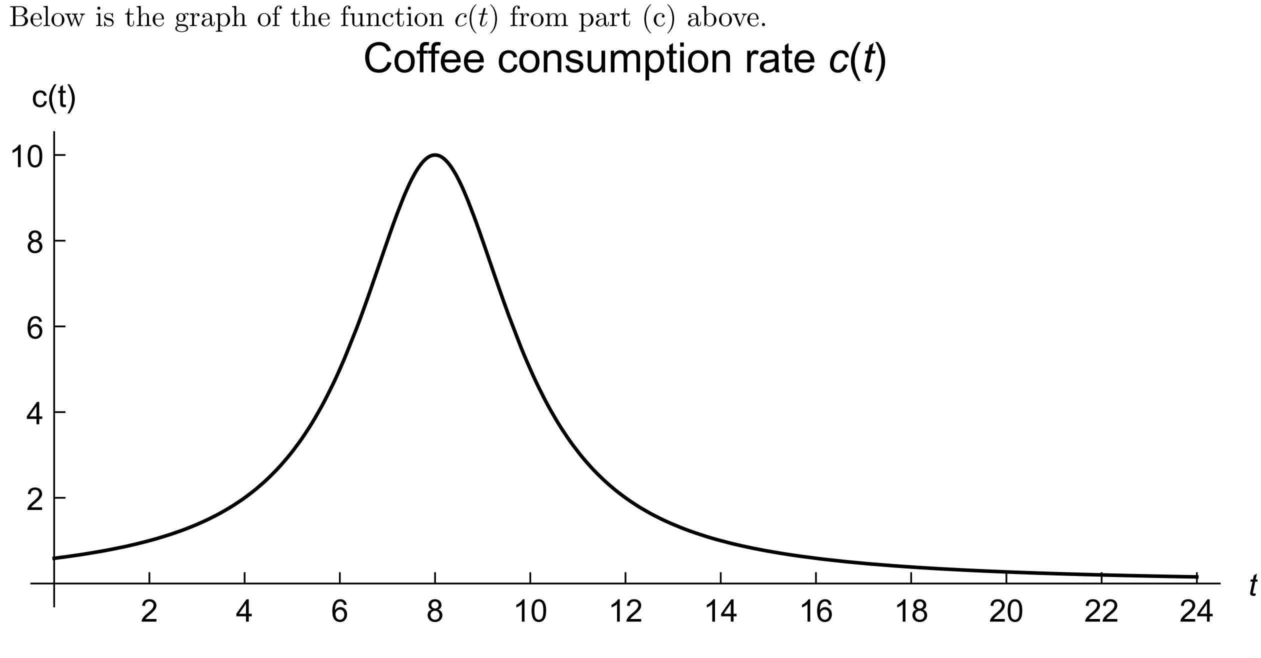
(b) Now suppose these students actually consume 8 gallons per hour between 6 am and 11 am, then consume 5 gallons per hour between 11 am and 4 pm, and finally they consume 3 gallons per hour between 4 pm and 11 pm. Calculate the total coffee consumed by this group between 6 am and 11 pm. Again, please express your answer as a whole number, and don't forget to include units.

(c) Suppose students consume coffee at a rate $c(t)$ given

$$c(t) = \frac{10}{1 + (.5t - 4)^2}$$

gallons per hour, where t is the number of hours since midnight.
Set up but DO NOT EVALUATE an integral to represent the total coffee consumed between 3 am and 6 pm. What are the units of the quantity represented by this integral?

(d) Below is the graph of the function $c(t)$ from part (c) above.



i. On the above graph, shade in the area that represents your answer to part (c) above.

ii. At what point in time, between 3 am and 6 pm, did CU students consume coffee most rapidly? By referring to the graph, please circle the correct answer, and explain.

3 am 11 am 8 am 6 pm 12 pm (noon)

iii. Using the graph above, determine whether the following is true or false: Between 3 am and 6 pm, CU students consumed more than 150 gallons of coffee. (Assume, again, that coffee is consumed at the rate $c(t)$ given by the above graph.) Please circle the correct answer, and explain.

TRUE FALSE