1. Using the table provided, find the following derivatives.

x	-1	0	1	2
$f(x)$	5	3	-4	2
$f'(x)$	-2	7	-3	2
$q(x)$	1	12	-9	10
$q'(x)$	-2	-4	-17	-6

(a) $g'(1)$, for $g(x) = f(x^3 - 2x^2 + 1)$

(b) $r'(2)$, for $r(x) = (f(x))^3 \cdot q(x)$

2. Find the indicated derivatives. You do not need to simplify your answers.

(a) $\frac{d}{dx}[3x^2 - x + 5]$

(b) $f'(z)$ if $f(z) = 3^z + \sqrt[3]{z} + \ln(3)$

(c) $h'(x)$ if $h(x) = 4\sin(e^x)$

(d) $\frac{dy}{dx}$ if $y = \frac{\tan(x)}{x+1}$

(e) $\frac{dp}{dt}$ if $p(t) = 2^{\cos(t)}$

(f) $\frac{dy}{dx}$ if $y = \cos(x^3)\ln(x + x^2)$

(g) $\frac{dy}{dx}$ if $y = \ln(\arctan(x))$

(h) $\frac{dy}{dx}$ if $y = \arctan(\ln(x))$

(i) $\frac{dy}{dx}$ if $y = \arctan(3x)\ln(4x)$

(j) $q'(2)$ if $q(x) = (1+r(x))^3$, $r(2) = 2$, and $r'(2) = 5$. Please express your answer as a whole number.

(k) $z'(0)$ if $z(t) = \arctan^3(r(t))$, $r(0) = 1$, and $r'(0) = -4$. Please express your answer in terms of π.

3. Let

$$g(x) = \sqrt{\frac{e^{x-2}}{e^{5x}}}.$$

Find $g'(-\frac{1}{2})$. **Hint:** Use properties of e^x to simplify *before* differentiating! Also, please simplify your final answer as much as possible.

4. For part (a) of this problem, find the derivative $\frac{dy}{dx}$ if

$$y = \sin(x^4)\tan(x^3 + x^2).$$

For part (b) of this problem, please state clearly which differentiation rules you used, and how many times you used each rule. (The possible rules are: Chain rule, product rule, quotient rule, sum rule, constant multiple rule.)

(a) $\frac{dy}{dx} =$

(b) Rules used, and how many times you used each rule:

5. Carbon 14 decays exponentially, with a per unit decay rate of 0.000121 year^{-1}. Suppose we start with 3000 mg of carbon 14. Here $C(t)$ is measured in milligrams and time t in years.

(a) Write an initial value problem for C, the amount of carbon 14 remaining at time t.

(b) Write a solution to the initial value problem you wrote in part (a).

(c) How much will be left after 500 years? Round to the nearest whole number, and please include units.

6. Simplify the following expressions as much as possible.

(a) $\ln(3x^2e^{2x})$

(b) $\ln\left(\frac{\sqrt{x^4+1}}{e^x}\right)$

(c) $\ln(e^{\ln(e^{10})})$

7. Consider the function $f(x) = \sqrt{2x-1}$.

(a) Write the microscope equation for $f(x)$, for arbitrary a and Δx. Your answer should be in terms of a and Δx alone.

(b) Adapt your answer to part (a) of this problem to approximate values of $f(x)$ near $x = 1$. Simplify as much as possible.

(c) Use your answer to part (b) of this problem to approximate $f(0.9)$ and $f(1.25)$.

(d) **Without using a calculator**, state which of your approximations in part (c) of this problem is closer to the actual value in question. Please explain how you know (again, without using a calculator) which one is better.

8. Find $f''(x)$ (the second derivative of $f(x)$, meaning the derivative of $f'(x)$) if

$$f(x) = xe^{2x}.$$

Carefully state which rules you used, and how many times you used each.

9. Use the microscope equation to estimate the value of $\dfrac{1}{2.1^2}$.

10. (a) Suppose $f(x)$ is a function for which $f(-1) = 3$ and $f'(-1) = -2$. Write down the microscope equation for f at $x = -1$.

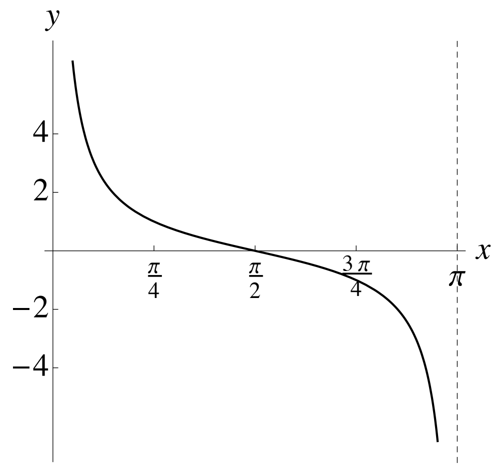
(b) Use the equation you wrote in part (a) to find an estimate for the value $f(-0.9)$.

(c) Without any additional information about the function f, which value would you expect your microscope equation to estimate better, $f(-2)$ or $f(-0.5)$? Explain your reasoning.

11. This problem involves the cotangent function $\cot(x)$, defined by

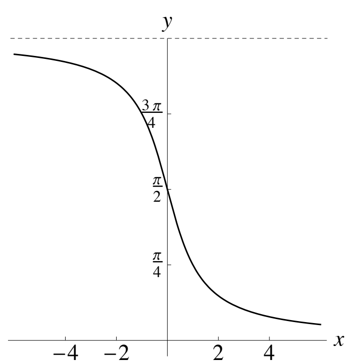
$$\cot(x) = \frac{\cos(x)}{\sin(x)}.$$

On the axes below is a graph of $y = \cot(x)$, for $0 < x < \pi$.

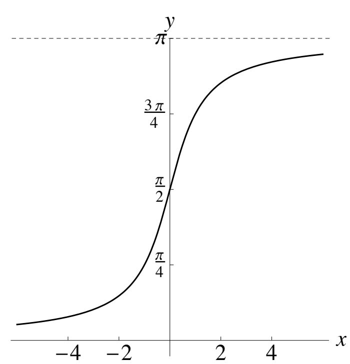


(a) Explain why reflecting $y = \cot(x)$, on this domain, about the line $y = x$ gives a new function, which we'll call $y = \text{arccot}(x)$.

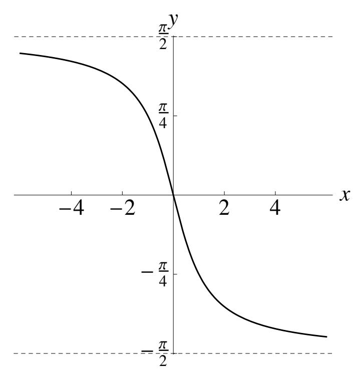
(b) Which of the following gives the graph of $y = \text{arccot}(x)$? Circle the letter ((A), (B), or (C)) below the correct graph.



(A)



(B)



(C)

(c) For this part of our problem, we will be using the fact (which is not hard to show) that

$$\frac{d}{dx}[\cot(x)] = -(1+\cot^2(x)).$$

Find $\frac{d}{dx}[\operatorname{arccot}(x)]$, as follows (fill in the blanks; there are five of them).
Since the function $y = \operatorname{arccot}(x)$ takes an input x to an output $\operatorname{arccot}(x)$, we know that the reflection $y = \cot(x)$ must take an input $\operatorname{arccot}(x)$ to an output x. That is,

$$\cot(\operatorname{arccot}(x)) = \underline{\qquad}. \tag{1}$$

We differentiate both sides of this equation to get

$$\frac{d}{dx}[\cot(\operatorname{arccot}(x))] = 1$$

or, using the chain rule and the fact that $d[\cot(x)]/dx = -(1+\cot^2(x))$ on the left,

$$-(1+\cot^2(\underline{\qquad\qquad})) \cdot \frac{d}{dx}[\underline{\qquad\qquad}] = 1. \tag{2}$$

Now again, $\cot(\operatorname{arccot}(x)) = x$ by (1), so equation (2) gives

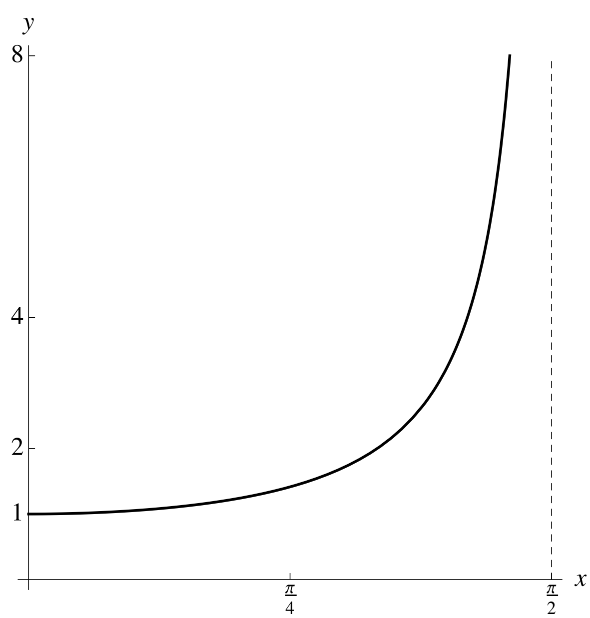
$$-(1+\underline{\qquad})\frac{d}{dx}[\operatorname{arccot}(x)] = 1$$

or, dividing by $-(1+x^2)$,

$$\frac{d}{dx}[\operatorname{arccot}(x)] = \underline{\qquad\qquad},$$

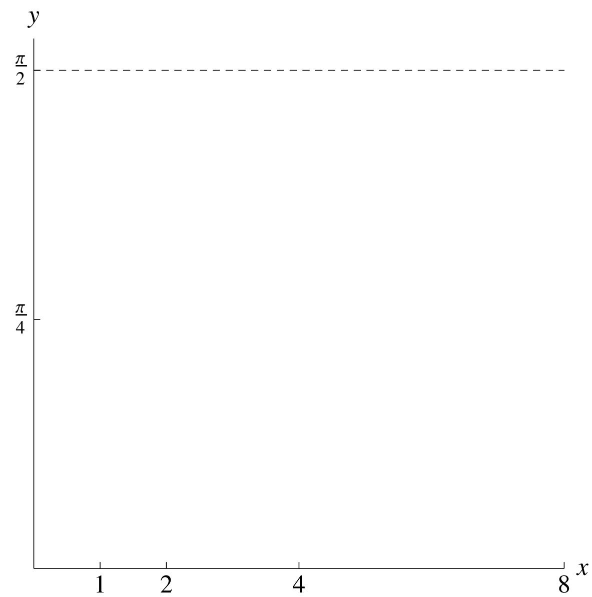
and we're done.

12. On the axes below is a graph of the function $y = \sec(x) = \dfrac{1}{\cos(x)}$, for $0, x, \pi/2$. Note that the dashed line is an asymptote; it's not actually part of the graph of the function.



(a) Explain why reflecting $y = \sec(x)$, on this domain, about the line $y = x$ gives a new function, which we'll call $y = \text{arcsec}(x)$.

(b) Sketch the graph of $y = \text{arcsec}(x)$, on the axes below.



(c) For this problem, you may use the fact that

$$\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x). \tag{$*$}$$

Find $\frac{d}{dx}[\text{arcsec}(x)]$, as follows (fill in the blanks; there are six of them).
Since the function $y = \text{arcsec}(x)$ takes an input x to an output $\text{arcsec}(x)$, we know that the reflection $y = \sec(x)$ must take an input $\text{arcsec}(x)$ to an ________ x. That is,

$$\sec(\text{arcsec}(x)) = \underline{\qquad}. \tag{1}$$

We differentiate both sides of this equation to get

$$\frac{d}{dx}[\sec(\text{arcsec}(x))] = 1$$

or, using the chain rule and (*) on the left,

$$\sec(\text{arcsec}(x))\tan(\underline{\qquad\qquad}) \cdot \frac{d}{dx}[\underline{\qquad\qquad}] = 1. \tag{2}$$

But for any real number θ between 0 and $\pi/2$, we have $\tan(\theta) = \sqrt{(\sec(\theta))^2 - 1}$. So

$$\tan(\text{arcsec}(x)) = \sqrt{(\sec(\text{arcsec}(x)))^2 - 1} = \sqrt{\underline{\qquad}^{\,2} - 1},$$

the last step by equation (1). Putting this, and equation (1), back into equation (2) gives

$$x\sqrt{x^2-1}\frac{d}{dx}[\text{arcsec}(x)] = 1$$

or, dividing by $x\sqrt{x^2-1}$,

$$\frac{d}{dx}[\text{arcsec}(x)] = \underline{\hspace{3cm}}.$$