1. Find the indicated derivatives. You DON'T have to simplify anything, or explain your steps.

(a) y' if $y = \pi^\pi$

(b) $\dfrac{dy}{dx}$ if $y = 3x^4 - \dfrac{4}{x^4} + \dfrac{3}{\sqrt[4]{x}}$

(c) y' if $y = \cos(4^x)$

(d) $\dfrac{d}{dx}[5\ln(\sin(x)) + 3\cos(\ln(x))]$

(e) $\dfrac{d}{dy}[\arctan(e^y)]$

(f) $h'(z)$ if $h(z) = \tan\big(z\cos(z)\big)$

2. Show that the given functions have the indicated derivatives. [You'll get substantial partial credit for differentiating correctly; but to get full credit, you must get your answer into the form shown.] Please JUSTIFY EACH STEP by citing the rule used, as shown in the following example.

EXAMPLE. Show that $\dfrac{d}{dx}\left[\dfrac{x\sin(x^2)}{\tan(x)}\right] = \dfrac{\tan(x)(2x^2\cos(x^2)+\sin(x^2)) - x\sin(x^2)\sec^2(x)}{\tan^2(x)}$.

SOLUTION.

$$\begin{aligned}
\frac{d}{dx}\left[\frac{x\sin(x^2)}{\tan(x)}\right] &= \frac{\tan(x)\,\frac{d}{dx}[x\sin(x^2)] - x\sin(x^2)\frac{d}{dx}[\tan(x)]}{\tan^2(x)} && \text{[quotient rule]}\\
&= \frac{\tan(x)(x\frac{d}{dx}[\sin(x^2)] + \sin(x^2)\frac{d}{dx}[x]) - x\sin(x^2)\cdot\sec^2(x)}{\tan^2(x)} && \text{[product rule]}\\
&= \frac{\tan(x)(x\cdot 2x\cos(x^2) + \sin(x^2)) - x\sin(x^2)\sec^2(x)}{\tan^2(x)} && \text{[chain rule]}\\
&= \frac{\tan(x)(2x^2\cos(x^2) + \sin(x^2)) - x\sin(x^2)\sec^2(x)}{\tan^2(x)}. && \text{[simplification]}
\end{aligned}$$

OK, here are the ones for you:

(a) Show that $\dfrac{d}{dx}\left[\dfrac{x\ln(x)}{1+e^x}\right] = \dfrac{(1+e^x)(1+\ln(x)) - x\ln(x)e^x}{(1+e^x)^2}$.

(b) Show that $\dfrac{d}{dx}[\cos(\sin(\cos(2x)))] = 2\sin(2x)\cos(\cos(2x))\sin(\sin(\cos(2x)))$.

3. (a) Find $\frac{d}{dx}\left[(1+\sqrt[3]{x})^3\right]$.

(b) Write down the microscope equation for $f(x) = (1+\sqrt[3]{x})^3$ near $x = 1$.

(c) Use your answer to part (b) above to estimate $(1+\sqrt[3]{1.05})^3$. Write out your answer to at least six decimal places. (If there are fewer than six decimal places in your result, fill out the remaining places with zeroes, e.g. 5.49=5.490000.) As a check on your work, you might want to take note that, according to a calculator, $(1+\sqrt[3]{1.05})^3 = 8.19837....$

4. A population grows exponentially, with per capita growth rate k.

(a) Write down an *initial value problem* (differential equation plus initial condition) that models the growth of this population, in terms of k and an initial population P_0.

(b) Suppose P is measured in thousands of individuals, and t in months. What are the units of k? Please explain.

(c) Suppose $P(0) = 700$ thousand individuals, and $P(1) = 710$ thousand individuals. Write down a formula for the population $P(t)$ after t months. In this formula, please express k to five decimal places.

(d) What is the population $P(t)$ after two months, to the nearest whole individual?

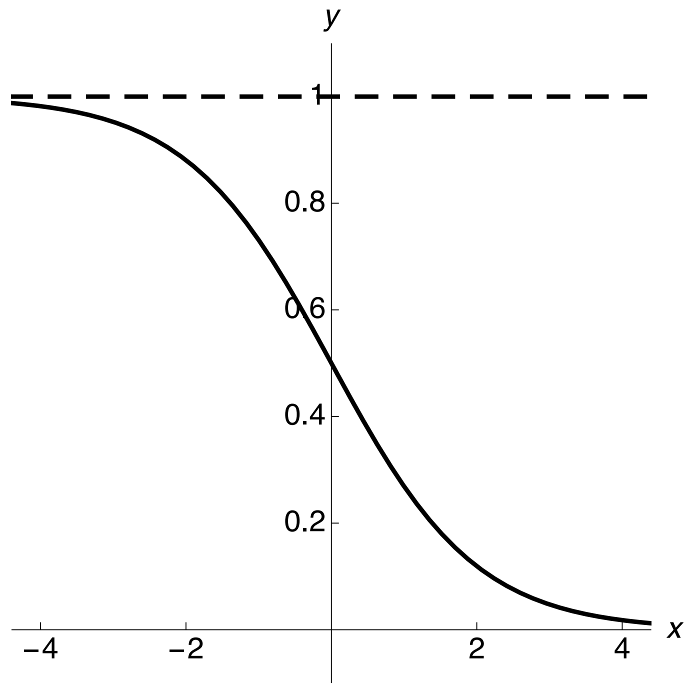
(e) What is the rate of growth dP/dt of the population after two months, to the nearest individual per month?

(f) How many months will it take for the population to reach 900 thousand individuals, to the nearest tenth of a month?

5. On the axes below is a graph of the function

$$\ell(x) = \frac{1}{1+e^x}.$$

The dashed line is an *asymptote* (a line that the graph of $\ell(x)$ gets closer and closer to); it's not part of the graph. The x axis is also an asymptote.



(a) Find $\ell'(x)$.

(b) Show that

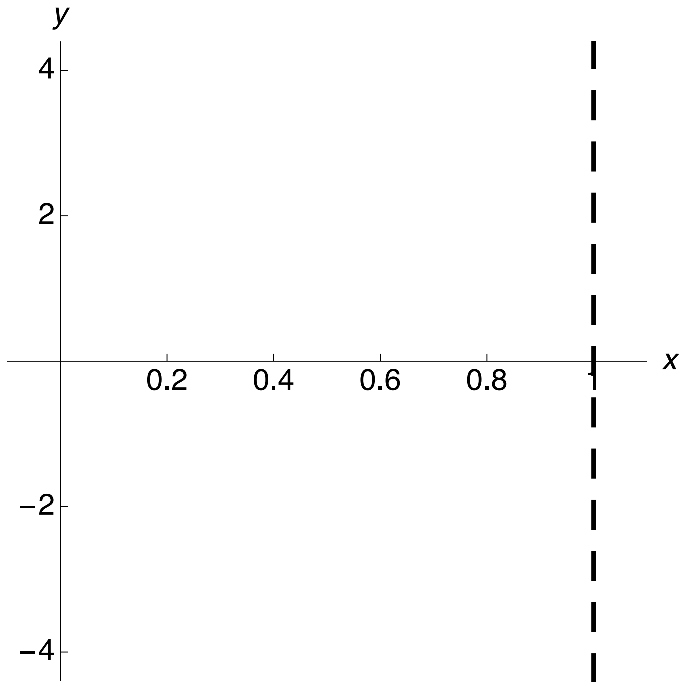
$$\ell'(x) = \ell^2(x) - \ell(x).$$

(Here, $\ell^2(x)$ denotes $(\ell(x))^2$.) Hint: Write out the right-hand side in terms of the above definition of $\ell(x)$, then get a common denominator.

We now wish to study the inverse function to $\ell(x)$, and to find the derivative of this inverse function.

(c) Explain why, when you flip (reflect) this function $y = \ell(x)$ about the line $y = x$, you get a new function, call it $y = q(x)$.

(d) Sketch the graph of $y = q(x)$, on the axes below.



(e) Fill in the blank (there's just one of them): Because the function $y = q(x)$ takes x to $q(x)$, we see that the flip $y = \ell(x)$ of $y = q(x)$ about the line $y = x$ takes $q(x)$ to _____; that is,

$$\ell(q(x)) = x. \tag{BLIP}$$

(f) Differentiate both sides of equation (BLIP), using the chain rule on the left hand side, to find a formula for $q'(x)$. Express your answer in terms of $\ell'(q(x))$.

(g) Use the result of parts (b) and (f) of this problem, and (BLIP), to show that

$$q'(x) = \frac{1}{x^2 - x}.$$

(h) What is the *slope* of the line tangent to the graph of $y = q(x)$ at $x = 1/2$? Use the previous part of this problem to answer. Please express your answer as a *whole number*, and draw this tangent line on your graph in part (b) above.