1. Find the following derivatives. You don't need to simplify or explain your steps.

   (a) $\dfrac{d}{dx}\left[\sqrt[4]{x} + \dfrac{1}{x^3} + \ln(x)\right]$

   (b) $\dfrac{d}{dx}\left[\dfrac{x^2 + x}{\sin(x)}\right]$

   (c) $\dfrac{d}{dx}\left[\dfrac{3x^4}{2} + \dfrac{x^3}{3} + x + e\right]$

   (d) $\dfrac{d}{dx}\left[\sin\left(\dfrac{\pi}{x^3}\right)\right]$

   (e) $\dfrac{d}{dx}\left[\tan\left(\sin\left(a^{x^2}\right)\right)\right]$

   (f) $\dfrac{d}{dz}\left[ae^b + \pi^y + z\right]$

2. Fill in each blank on the right hand side with the name of the rule that is being used there: **Constant Multiple, Sum, Chain, Product, Quotient, or None.**

   At most one rule is being used per line. (In "real life," you might want to combine some steps.) If *none* of the rules listed above are being used, then write "none."

$$\frac{d}{dx}\left[4x^3 \ln\left(\frac{\sin x}{x+x^3}\right)\right] = \ln\left(\frac{\sin x}{x+x^3}\right)\frac{d}{dx}\left[4x^3\right] + 4x^3\frac{d}{dx}\left[\ln\left(\frac{\sin x}{x+x^3}\right)\right] \qquad (\underline{\hspace{3cm}})$$

$$= \ln\left(\frac{\sin x}{x+x^3}\right)\cdot 4\frac{d}{dx}\left[x^3\right] + 4x^3\frac{d}{dx}\left[\ln\left(\frac{\sin x}{x+x^3}\right)\right] \qquad (\underline{\hspace{4cm}})$$

$$= 12x^2 \ln\left(\frac{\sin x}{x+x^3}\right) + 4x^3\frac{d}{dx}\left[\ln\left(\frac{\sin x}{x+x^3}\right)\right] \qquad (\underline{\hspace{1.5cm}})$$

$$= 12x^2 \ln\left(\frac{\sin x}{x+x^3}\right) + 4x^3\cdot\frac{x+x^3}{\sin x}\frac{d}{dx}\left[\frac{\sin x}{x+x^3}\right] \qquad (\underline{\hspace{2.5cm}})$$

$$= 12x^2 \ln\left(\frac{\sin x}{x+x^3}\right) + 4x^3\cdot\frac{x+x^3}{\sin x}\cdot\frac{(x+x^3)\frac{d}{dx}\left[\sin x\right] - \sin x\frac{d}{dx}\left[x+x^3\right]}{(x+x^3)^2} \qquad (\underline{\hspace{3cm}})$$

$$= 12x^2 \ln\left(\frac{\sin x}{x+x^3}\right) + \frac{4x^4+4x^6}{\sin x}\cdot\frac{(x+x^3)\cos x - \sin x\frac{d}{dx}\left[x+x^3\right]}{(x+x^3)^2} \qquad (\underline{\hspace{1.5cm}})$$

$$= 12x^2 \ln\left(\frac{\sin x}{x+x^3}\right) + \frac{4x^4+4x^6}{\sin x}\cdot\frac{(x+x^3)\cos x - \sin x\left(\frac{d}{dx}\left[x\right] + \frac{d}{dx}\left[x^3\right]\right)}{(x+x^3)^2} \qquad (\underline{\hspace{2.5cm}})$$

$$= 12x^2 \ln\left(\frac{\sin x}{x+x^3}\right) + \frac{4x^4+4x^6}{\sin x}\cdot\frac{(x+x^3)\cos x - \sin x\left(1 + \frac{d}{dx}\left[x^3\right]\right)}{(x+x^3)^2} \qquad (\underline{\hspace{1.5cm}})$$

$$= 12x^2 \ln\left(\frac{\sin x}{x+x^3}\right) + \frac{4x^4+4x^6}{\sin x}\cdot\frac{(x+x^3)\cos x - \sin x\left(1 + 3x^2\right)}{(x+x^3)^2} \qquad (\underline{\hspace{1.5cm}})$$

3. For each of the following, SHOW, by direct computation, that the indicated function has the indicated derivative. State which rules you used, and how many times you used each.

   (a) $y = \ln(e^x)$; $y' = 1$.

   (b) $y = \ln(\ln(2x))$; $y' = \dfrac{1}{x\ln(2x)}$.

   (c) $y = \tan(2z) - 2z$; $\dfrac{dy}{dz} = 2\tan^2(2z)$. (You may use the trig identity $\tan^2\theta = \sec^2\theta - 1$.)

   (d) $r = \ln^3(q^2+1)$; $\dfrac{dr}{dq} = \dfrac{6q\ln^2(q^2+1)}{q^2+1}$

4. Use the Microscope Equation to approximate $\sqrt{4.1}$.

5. Let $f(x) = \ln(1 + x)$.
   (a) Write down the Microscope Equation for $f(x)$ at $x = 0$.
   (b) Use your answer from part (a) to approximate $\ln(1.01)$ and $\ln(0.99)$.

6. If $g(x) = x^2$, write down the Microscope Equation for $g(x)$ at the point $x = 2$. Use this to approximate $g(2.01)$ and $g(2.8)$. Which do you think is a better estimate?