1. Given the following three functions, find the values and expressions requested below. Please simplify your answers.

$$f(x) = 2 - 3x, \qquad g(x) = \frac{2}{x+1}, \qquad h(x) = x^2 - x$$

   (a) $f(g(0)) =$

   (b) $g(h(x)) =$

   (c) $h(f(x)) =$

   (d) $h(f(g(-2))) =$

2. (a) Write the equation of the line $y = f(x)$ through the points (3,4) and (5,8). Please write your final answer in slope-intercept form.

   (b) What is (are) the y-intercept(s) of this line?

   (c) What is (are) the x-intercept(s) of this line?

   (d) Write the equation of the line $y = g(x)$ through the points $(-1,1)$ and $(1,-3)$. Please write your final answer in slope-intercept form.

   (e) What are the x and y coordinates of the point(s) where the lines from (a) and (d) intersect (if they intersect at all)?

3. Determine derivatives of the following functions. (You do NOT need to use the definition of the derivative for these.) You do NOT need to simplify your answers.

   (a) $f(x) = 3x^4 + x^{1/3} - \dfrac{1}{\sqrt{x}} - 2\sin(x)$

   (b) $g(x) = \dfrac{2 + x^3}{x}$

   (c) $k(r) = 5^r$

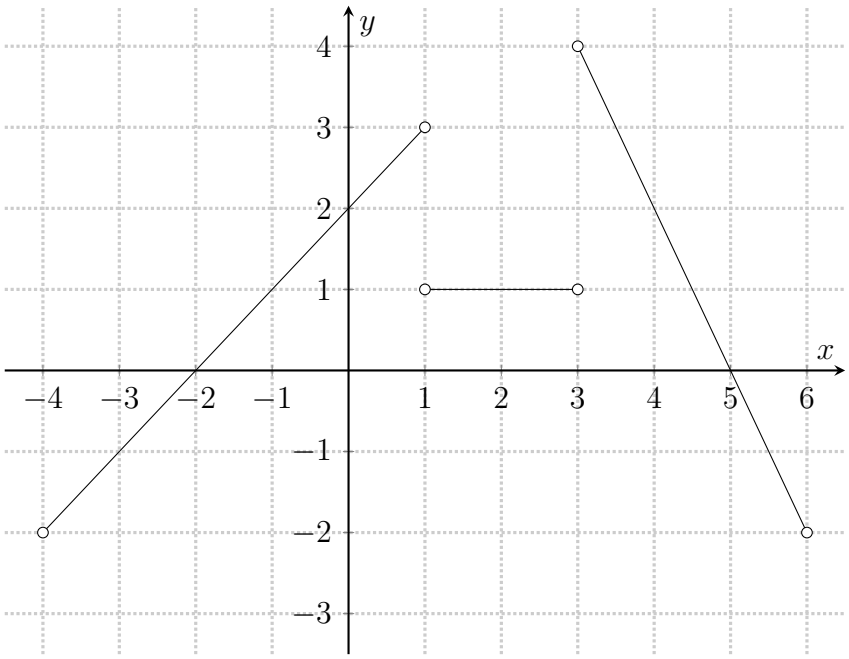
   (d) $\dfrac{d}{dx}\left[\pi^2\right]$

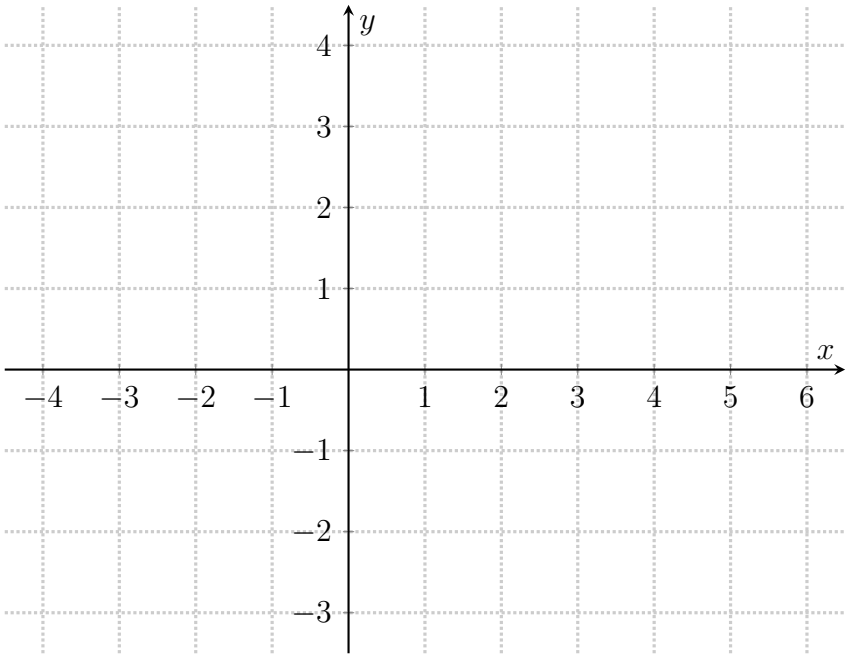
4. Consider the function

$$g(x) = \begin{cases} 2x + 1, & x \geq 0 \\ 2x - 1, & x < 0 \end{cases}$$

   Is $g(x)$ locally linear (differentiable) at $x = 0$? If yes, what is $g'(0)$? If no, why is $g'(x)$ not locally linear (not differentiable) at $x = 0$?

5. Consider the function $h(x)$ drawn below. Sketch the graph of $h'(x)$ on the empty axes.





6. The rate of change of salt dissolving in water is given by the following equation:

$$S' = -\frac{3}{5}S$$

where S is the amount of salt remaining at a particular time. Assume that salt is measured in grams and time is measured in minutes.

(a) What are the units for S'?

(b) Assume that $S(0) = 60$g. Use Euler's method to estimate how much salt will remain undissolved after 3 minutes, using a step size of 1.5 minutes. Include units.

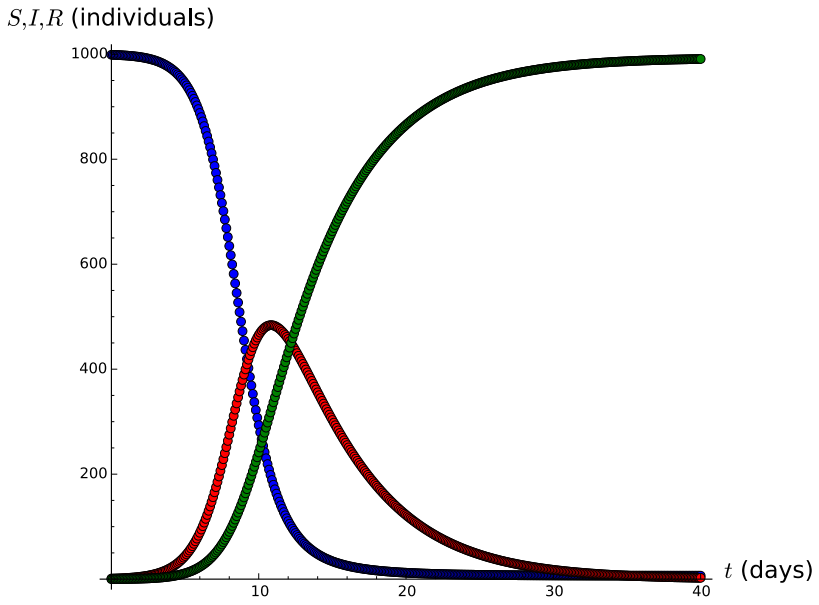
(c) Calculate the same estimate indicated in part (b), but with a step size of 1 minute. Include units.

(d) Consider your answers from parts (b) and (c) of this problem. Which gives a more accurate estimate of the amount of undissolved salt in the water after 3 minutes? Explain your answer in terms of step size and rates of change.

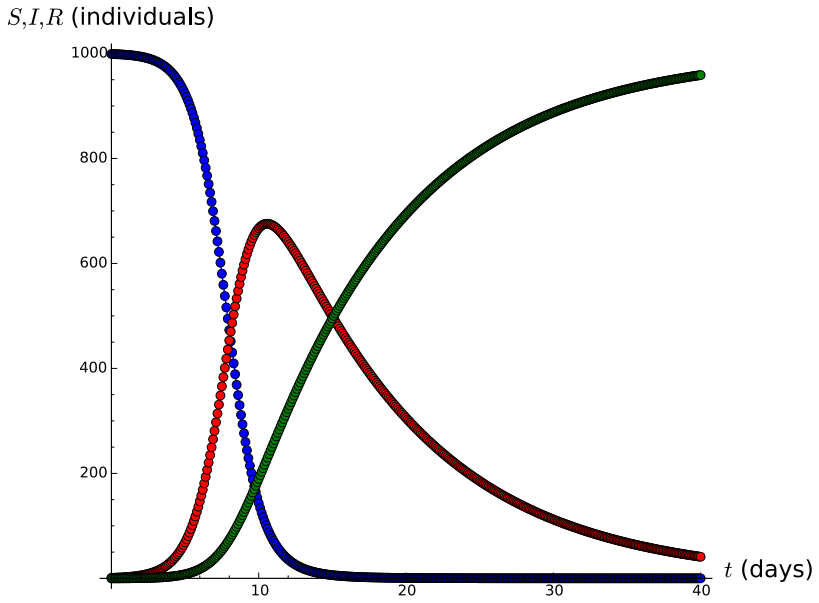
7. Below are graphs of S, I, and R for two different epidemics, evolving according to the usual SIR equations:

$$\boxed{\begin{aligned} S' &= -a\,S\,I \\ I' &= a\,S\,I - b\,I \\ R' &= \ b\,I \end{aligned}}$$

Epidemic 1:



Epidemic 2:



The following questions refer to the graphs above.

(a) On each graph, label which curve represents S (susceptible), which represents I (infected), and which represents R (recoverded).

(b) Recall that, for a disease that evolves according to the SIR model, the *threshold value* S_T of S is the value of S where I peaks; that is, it's the value of S where $I' = 0$.

Using the graphs, estimate the threshold value S_T of S for each epidemic. Please show your work, by drawing appropriate lines on the above graphs, and/or explain your answers. Include units in your answers.

Epidemic 1: $S_T =$ _____

Epidemic 2: $S_T =$ _____

(c) Both of these epidemics have the same starting conditions and the same transmission coefficient a. However, one disease lasts five days, while the other lasts ten days. Which epidemic has the disease of **longer** duration, in other words, the disease where it takes **10 days** to recover? *Circle one and explain your answer.*

Epidemic 1 Epidemic 2

8. Let $f(x) = x^2 + 3x + 1$.

(a) Find the average rate of change $\dfrac{\Delta y}{\Delta x}$ of $f(x)$ with respect to x, from $x = 2$ to $x = 2.1$.

(b) Find the average rate of change $\dfrac{\Delta y}{\Delta x}$ of $f(x)$ with respect to x, from $x = 2$ to $x = 2.01$.

(c) Show that the average rate of change $\dfrac{\Delta y}{\Delta x}$ of $f(x)$ with respect to x, from $x = 2$ to $x = 2 + \Delta x$, equals $7 + \Delta x$.

(d) Use the definition of the derivative (the second item on your formula sheet) to find $f'(2)$.

(e) Find the equation of the tangent line to $f(x)$ at $x = 2$.

9. Suppose the equation of the tangent line to a function $f(x)$ at $x = 4$ is given by $y = \frac{1}{2}x+3$.

(a) Find $f'(4)$.

(b) Find $f(4)$.