1. The two points $(-1, -3)$ and $(2, -9)$ lie on a line.

   (a) What is the slope of the line through $(-1, -3)$ and $(2, -9)$?

   (b) What is the equation (in any form) of the line through $(-1, -3)$ and $(2, -9)$?

   (c) What is (are) the y-intercept(s) of the line you found in part (b)?

   (d) What is (are) the x-intercept(s) of the line you found in part (b)?

2. Consider the function $f(x) = x^3 + 3^x$. What is the equation of the tangent line to $f(x)$ at $x = 1$?

3. Find the indicated derivatives.

   (a) $f'(x)$ if $f(x) = 77x^{10} - 10x^{77} - x^{10/7} + \dfrac{10}{7x^{7/10}} - \pi^7$.

   (b) $\dfrac{d}{dx}\left[3\cos(x) - \dfrac{\tan(x)}{3} + 3^x - \ln(3)\right]$.

   (c) $\dfrac{d}{dz}[y^z]$.

   (d) $\dfrac{d}{dq}[abq^2 + cdq - 4]$.

4. Suppose a population of chinchillas changes with time (where time is measured in years) according to the following rate equation:

$$C' = (1.14)C.$$

   (a) Is the population of chinchillas increasing or decreasing? Explain your answer.

   (b) Assume that the chinchilla population begins with 100 individuals. Use Euler's method to estimate the size of the population after 6 years, using a step size of 2 years.

   (c) What could you change about your calculations in part (b) to improve your estimate?

5. Nestled in the mountains of Syrinx, on the banks of the river Alph, is a town called Ixalan. Ixalan has a population of 100,000 Garruks. A certain mysterious disease (rumored to be contracted from demogorgons) is spreading through this Garruk population, according to the usual SIR equations:

$$\begin{aligned} S' &= -aSI, \\ I' &= aSI - bI, \\ R' &= \phantom{-aSI - {}} bI. \end{aligned}$$

Here S, I, and R denote the number of Garruks susceptible, infected, and recovered, respectively, at any given time t. We agree that t is measured in days, and that S, I, and R are measured in individual Garruks.

It's known that, for this particular disease, it takes a Garruk 20 days on average to recover. (That is, on average one stays infected for 20 days.)

It's also observed that, after 5 days, there are 96,605 Garruks still susceptible and 2,988 Garruks infected – that is, $S(5) = 96{,}605$ and $I(5) = 2{,}988$. It's also noted that, one half of a day later, there are 96,172 Garruks still susceptible and 3,346 Garruks infected – that is, $S(5.5) = 96{,}172$ and $I(5.5) = 3{,}346$.

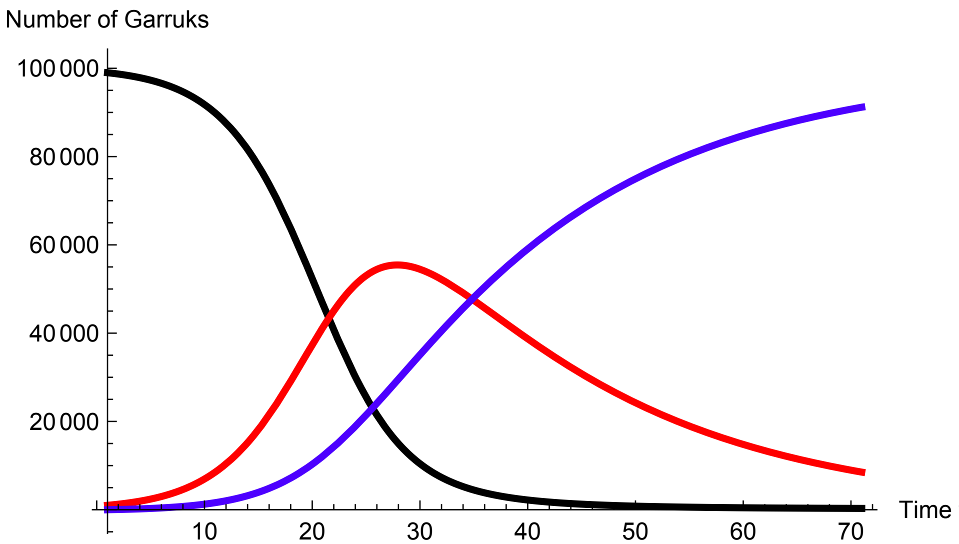
Assume throughout this problem that the total number of Garruks always remains the same.

(a) Using information given above, find the average rate of change of the susceptible population from $t = 5$ to $t = 5.5$. Please include units in your answer.

(b) Find the approximate value of the transmission coefficient a. Hint: use your answer to part (a) of this problem, above, as an approximation to $S'(5)$. Then use one of the SIR equations. Please write your answer to *at least six* decimal places, and include units in your answer.

(c) Using your answer to part (b) above, and additional information given in this problem, find the approximate value of S_T, the threshold value of S, to the nearest whole Garruk.

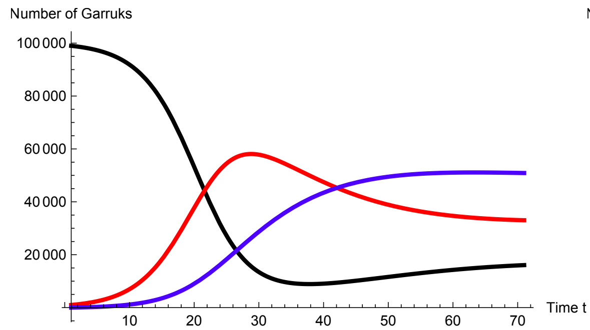
(d) Now approximate S_T in a different way, namely: by reading it off of the graph below. (A rough, eyeball estimate is fine.) Please show your work, by drawing appropriate line(s) on the graph to reflect your reasoning, and/or explain your answer. (Note: your answer might be somewhat different from your answer to part (c) above; that's the thing about estimates.) Again, please include units in your answer.

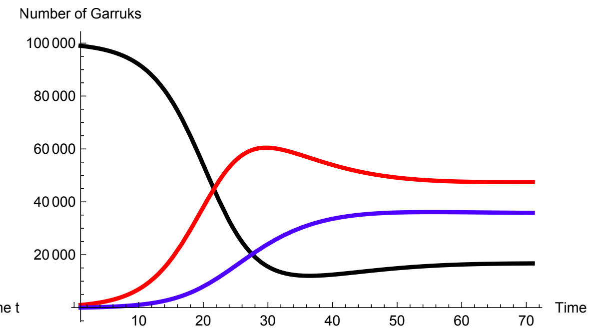


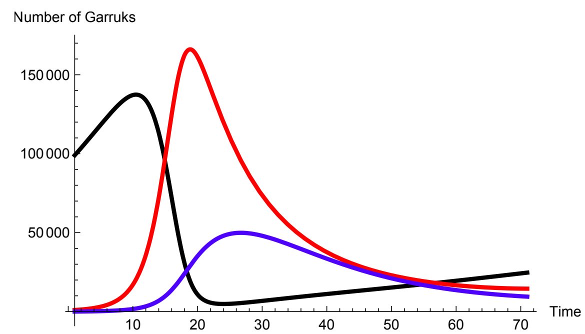
$S_T \approx$ ________________

(e) Modify the above SIR equations for this problem to model a situation where recovered become susceptible again after 15 days. (Your new equations should have actual values written in for the parameters, not just letters like a, b, etc.)

(f) Which of the three graphs below corresponds to the situation you modeled in the previous part of this problem? Please explain. Hint: compare duration of infection to duration of recovery.







6. Use the definition of the derivative to find $f'(9)$ for $f(x) = \sqrt{x}$. Hint: multiply $\dfrac{\sqrt{9+\Delta x}-3}{\Delta x}$ by $\dfrac{\sqrt{9+\Delta x}+3}{\sqrt{9+\Delta x}+3}$. Then do some algebra until you can cancel a Δx top and bottom.