The Golden Ratio!!!!

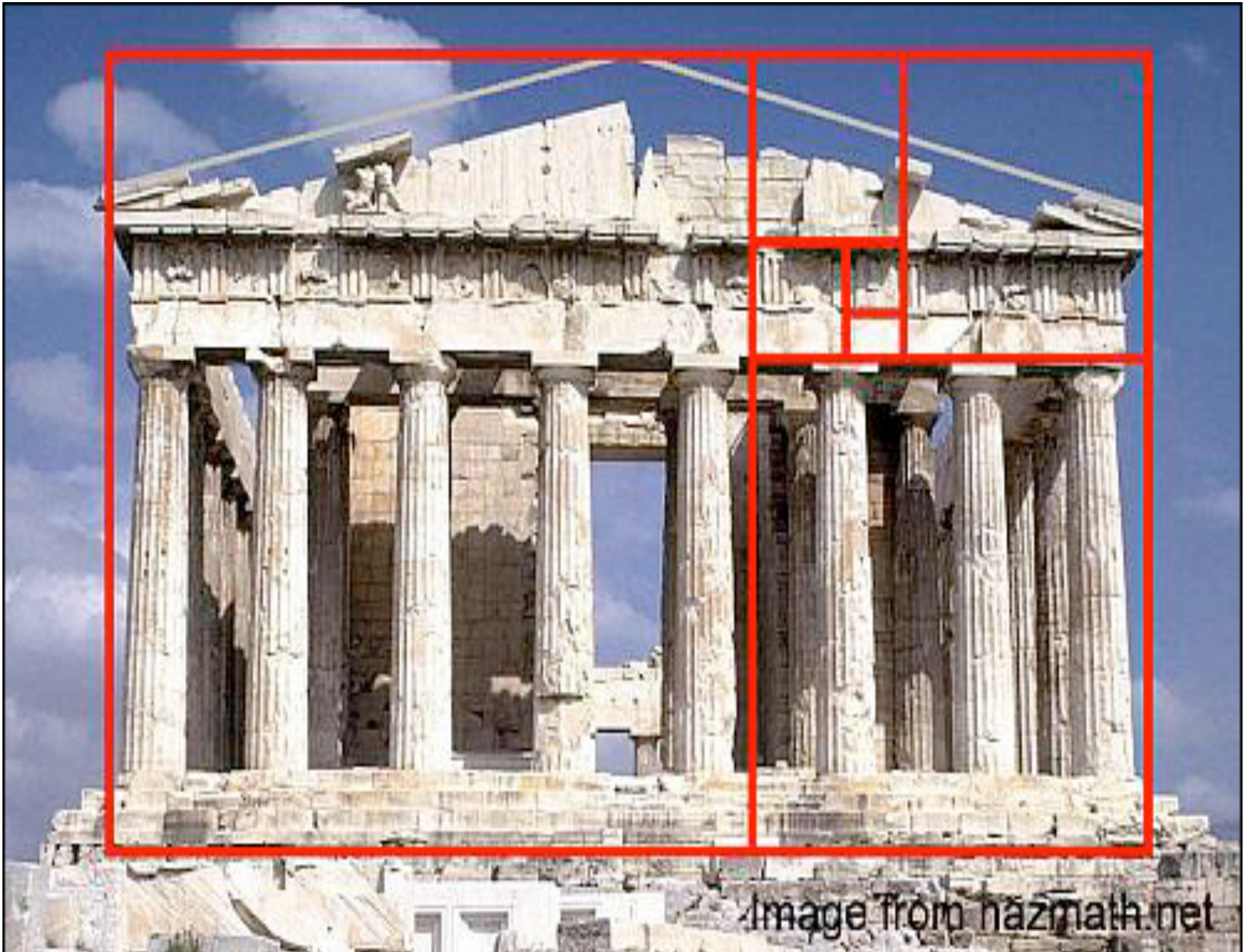
Image from nazmath.net


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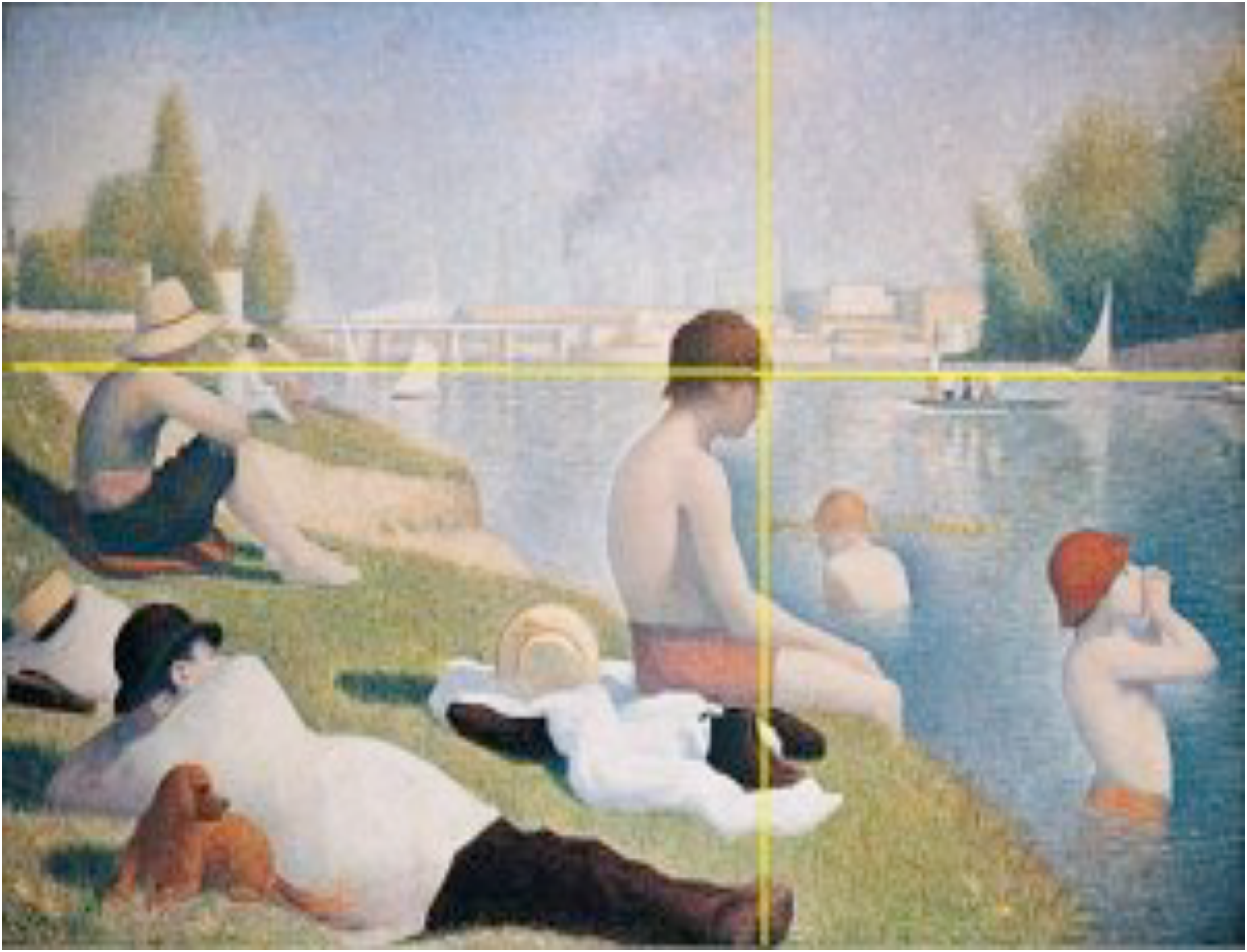
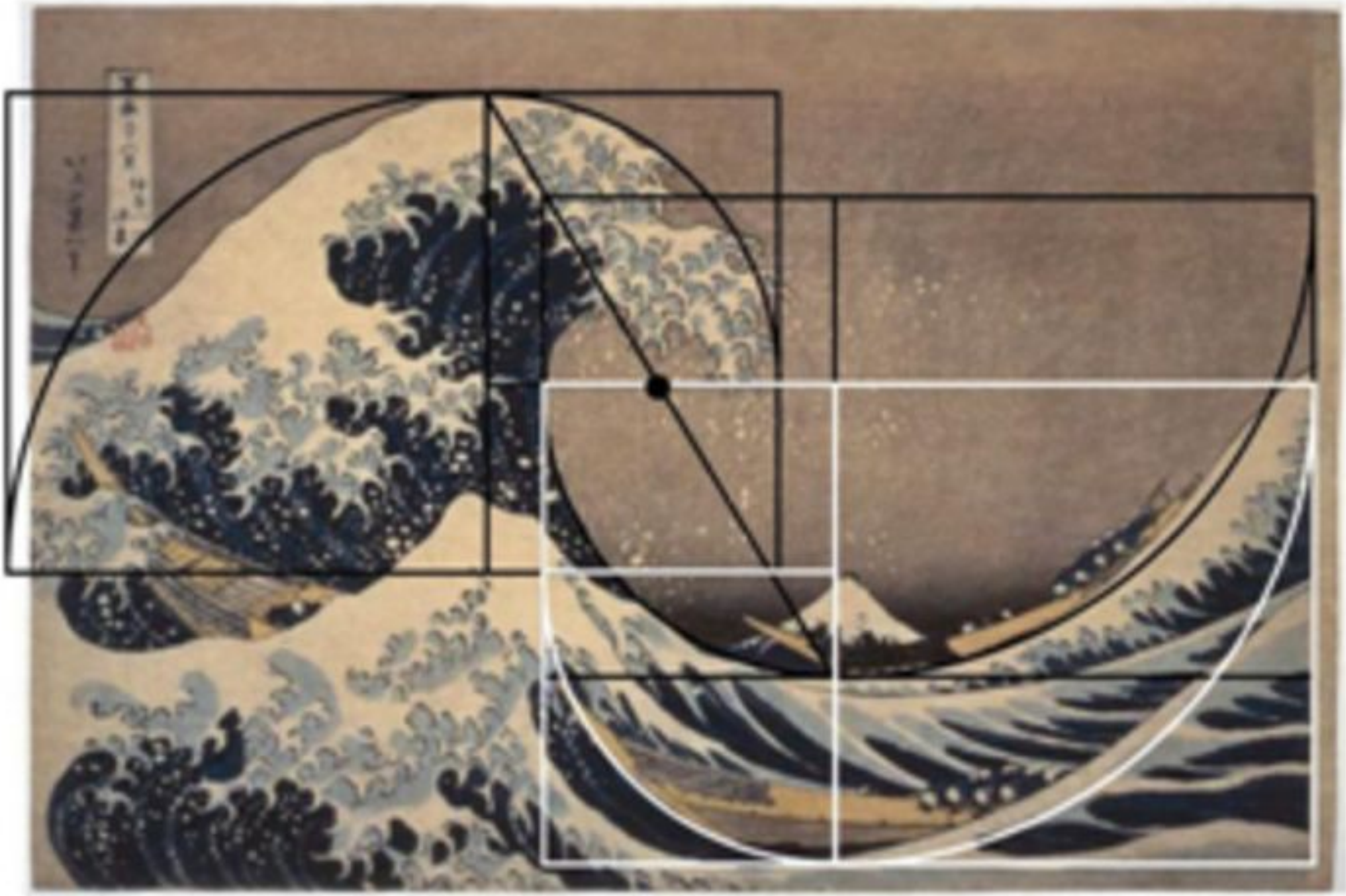
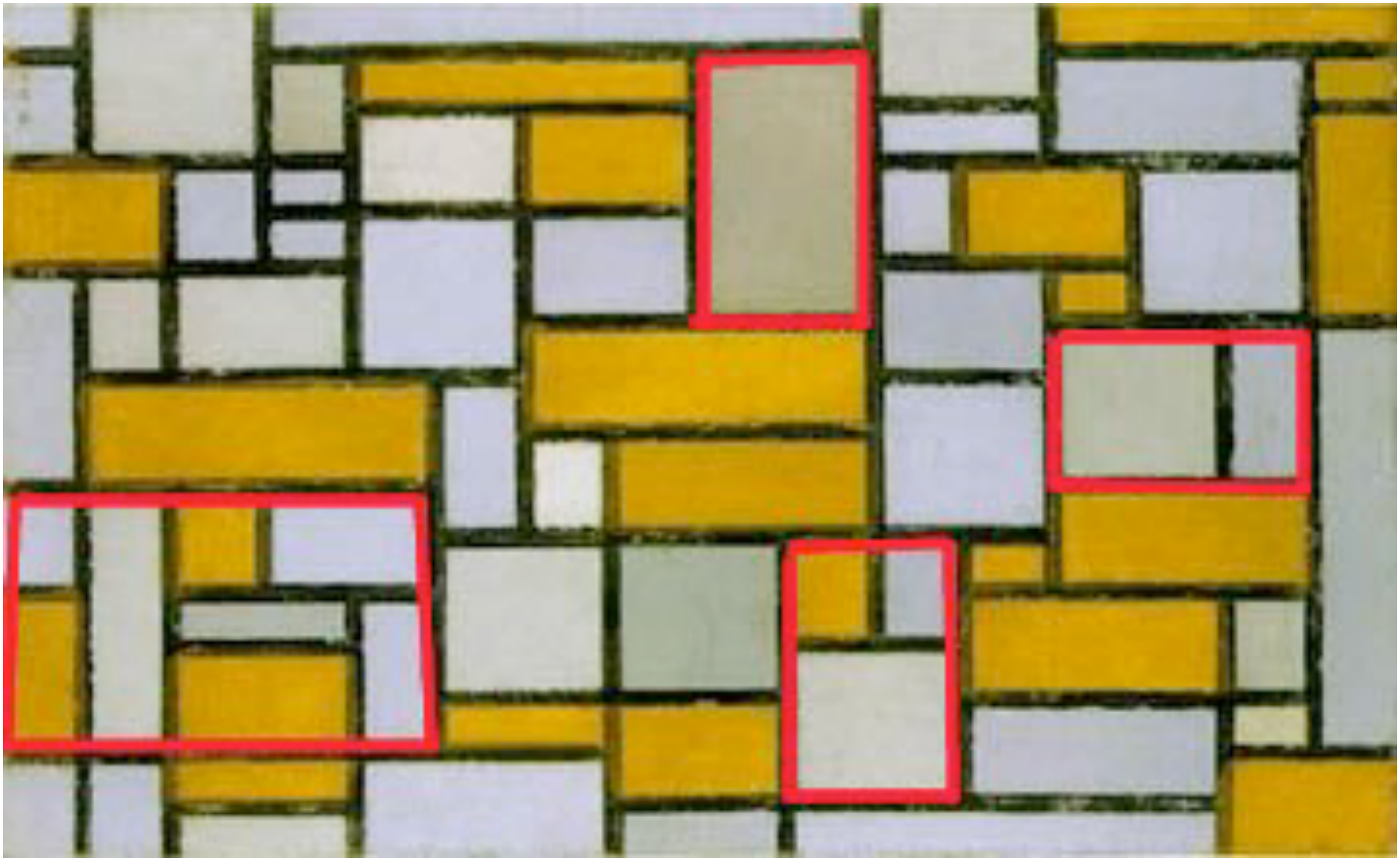
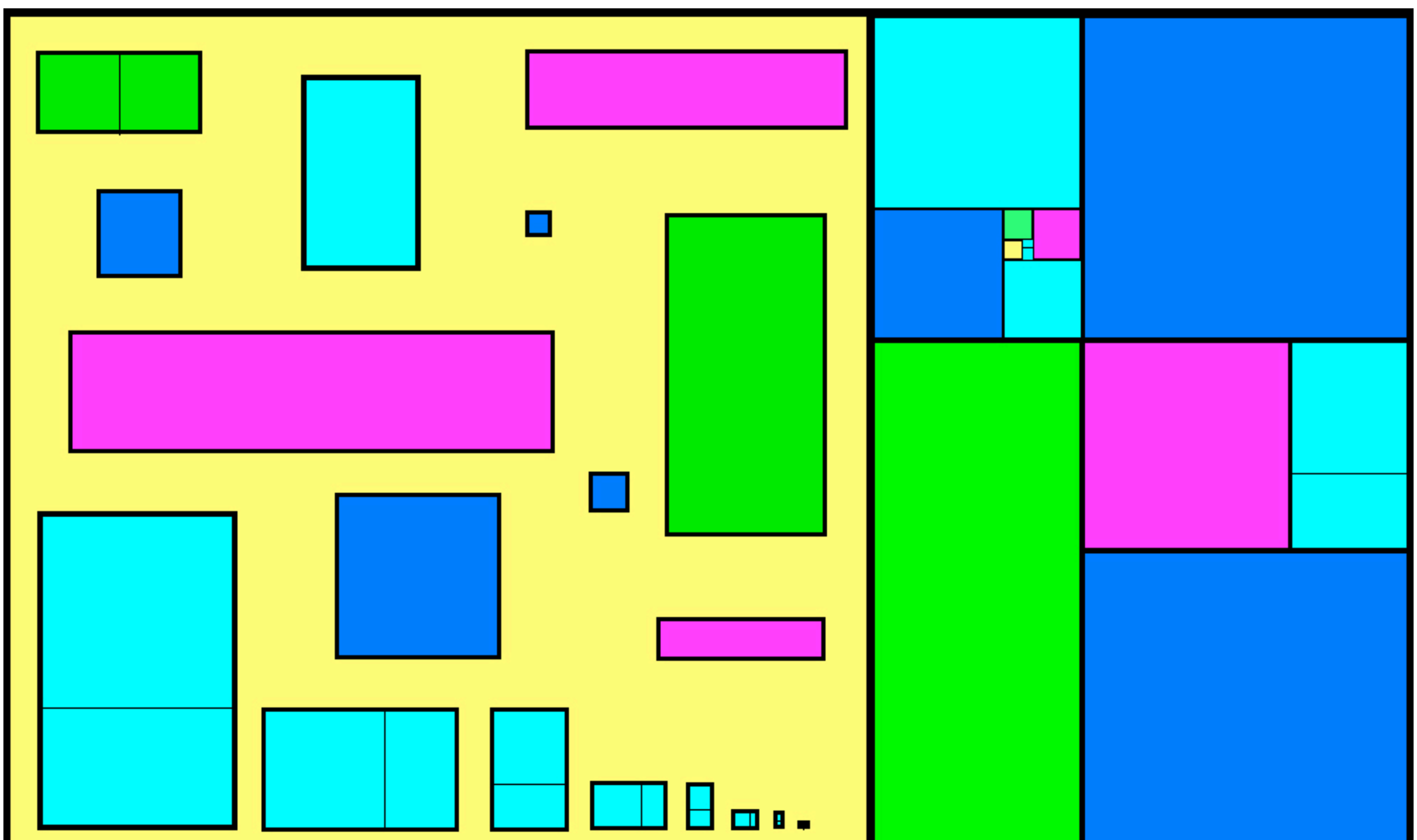
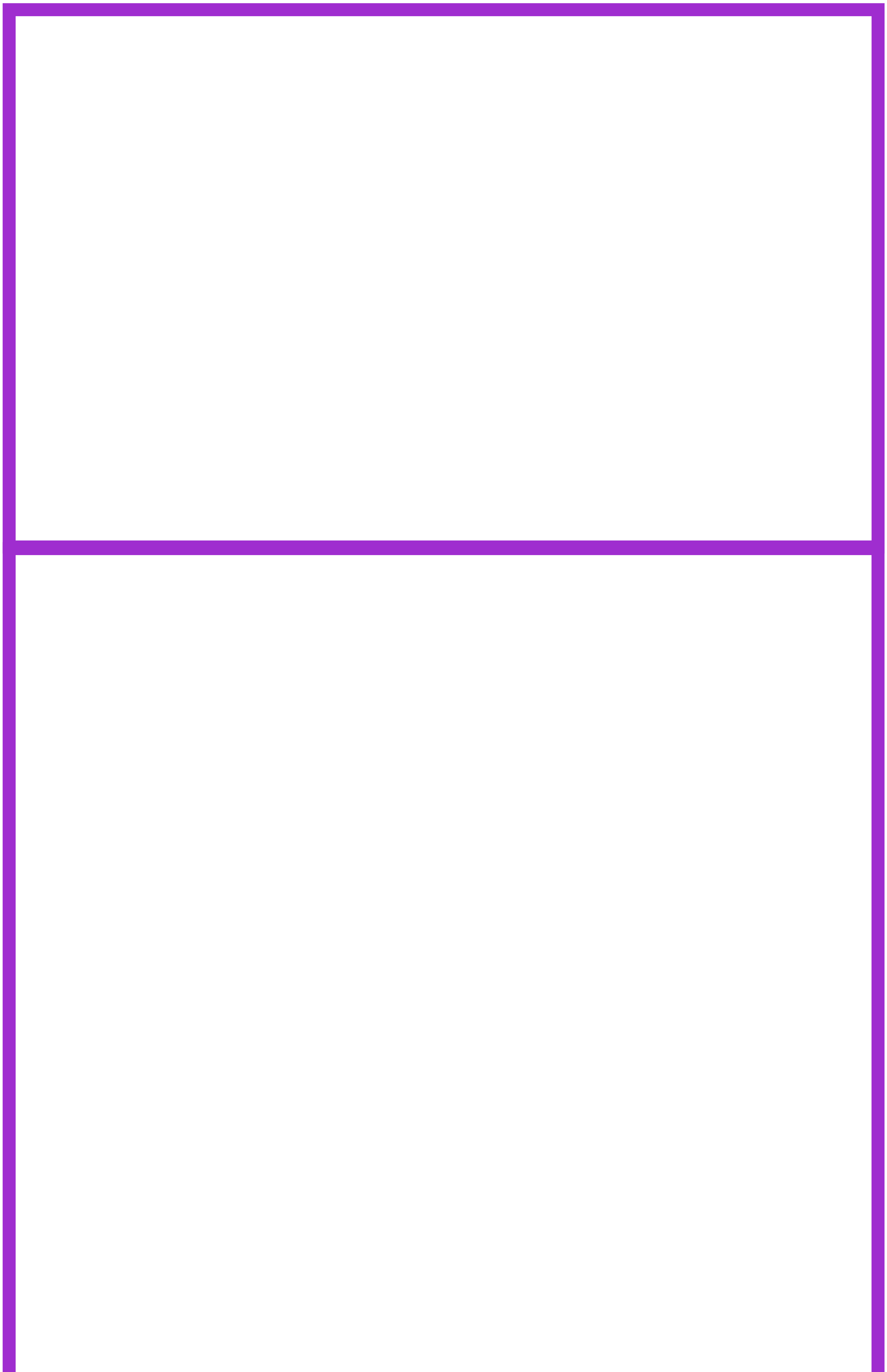
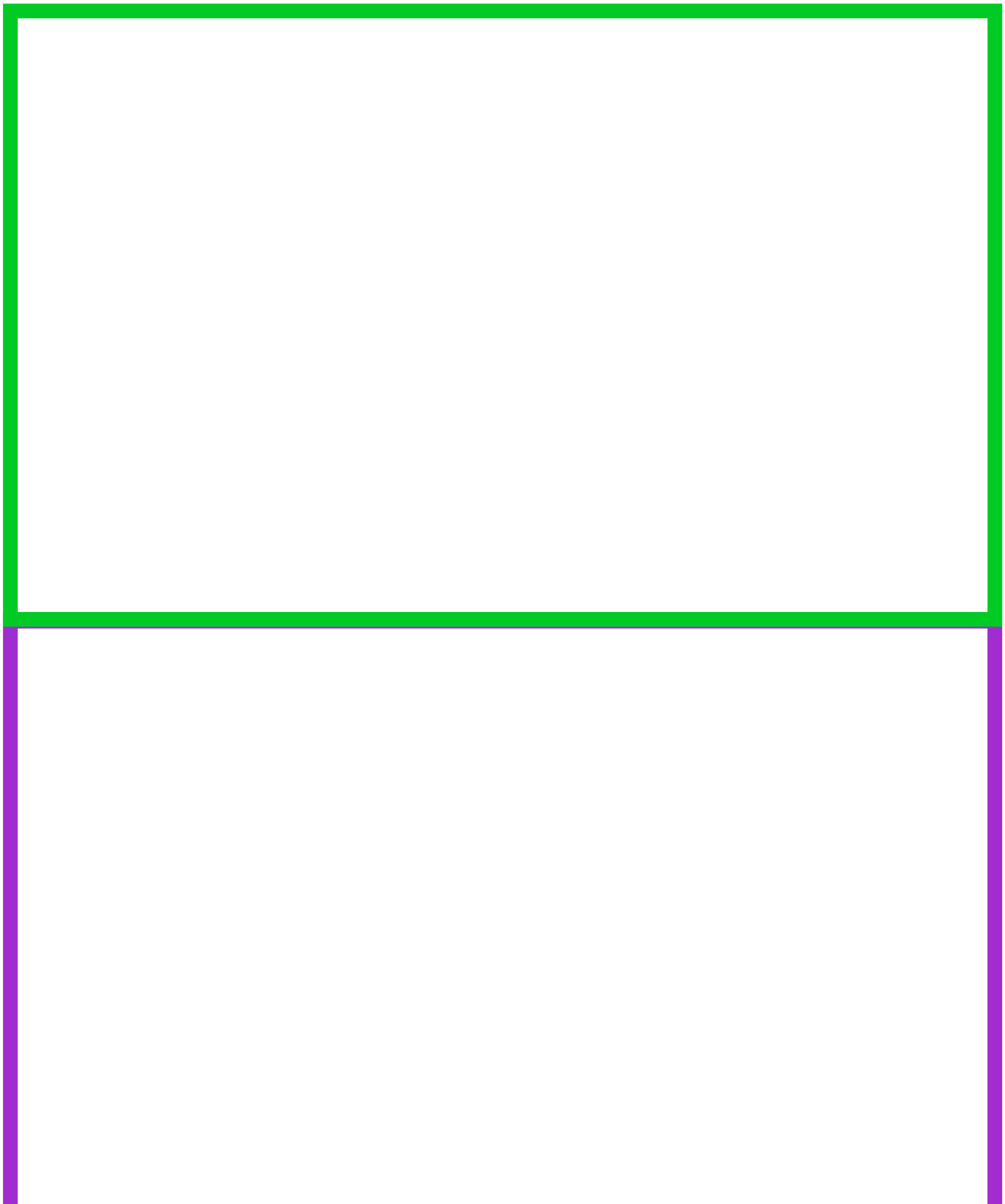


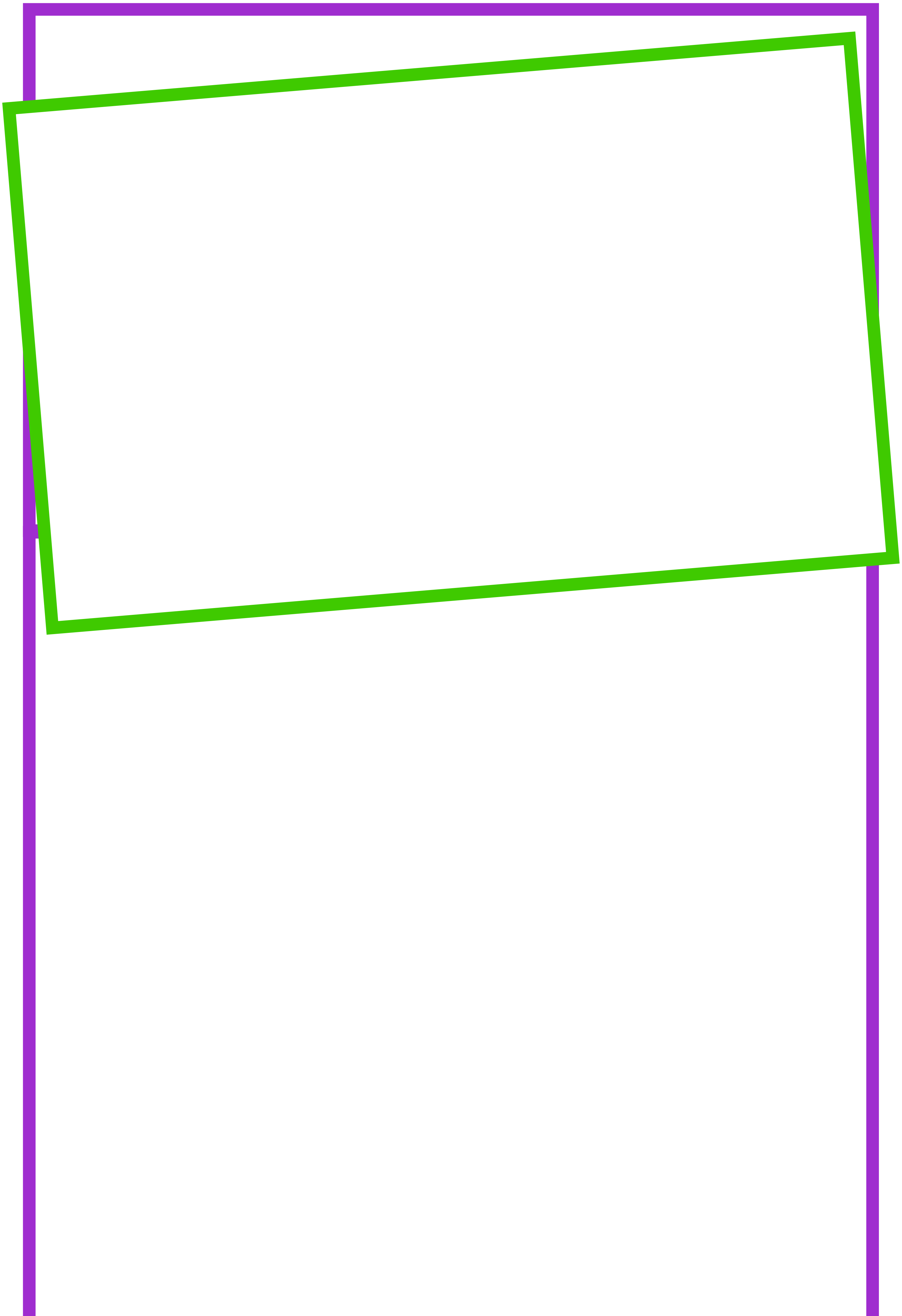


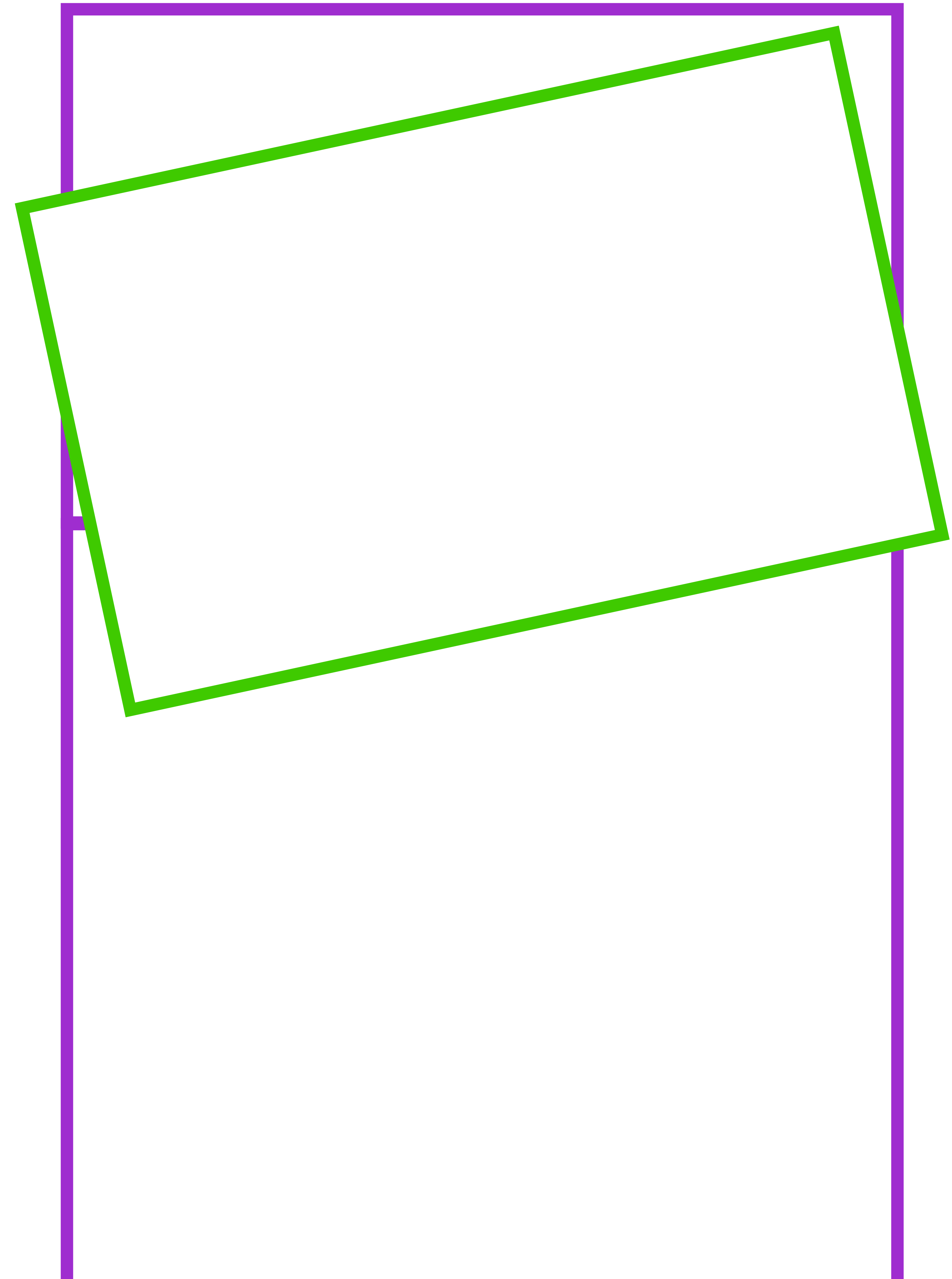


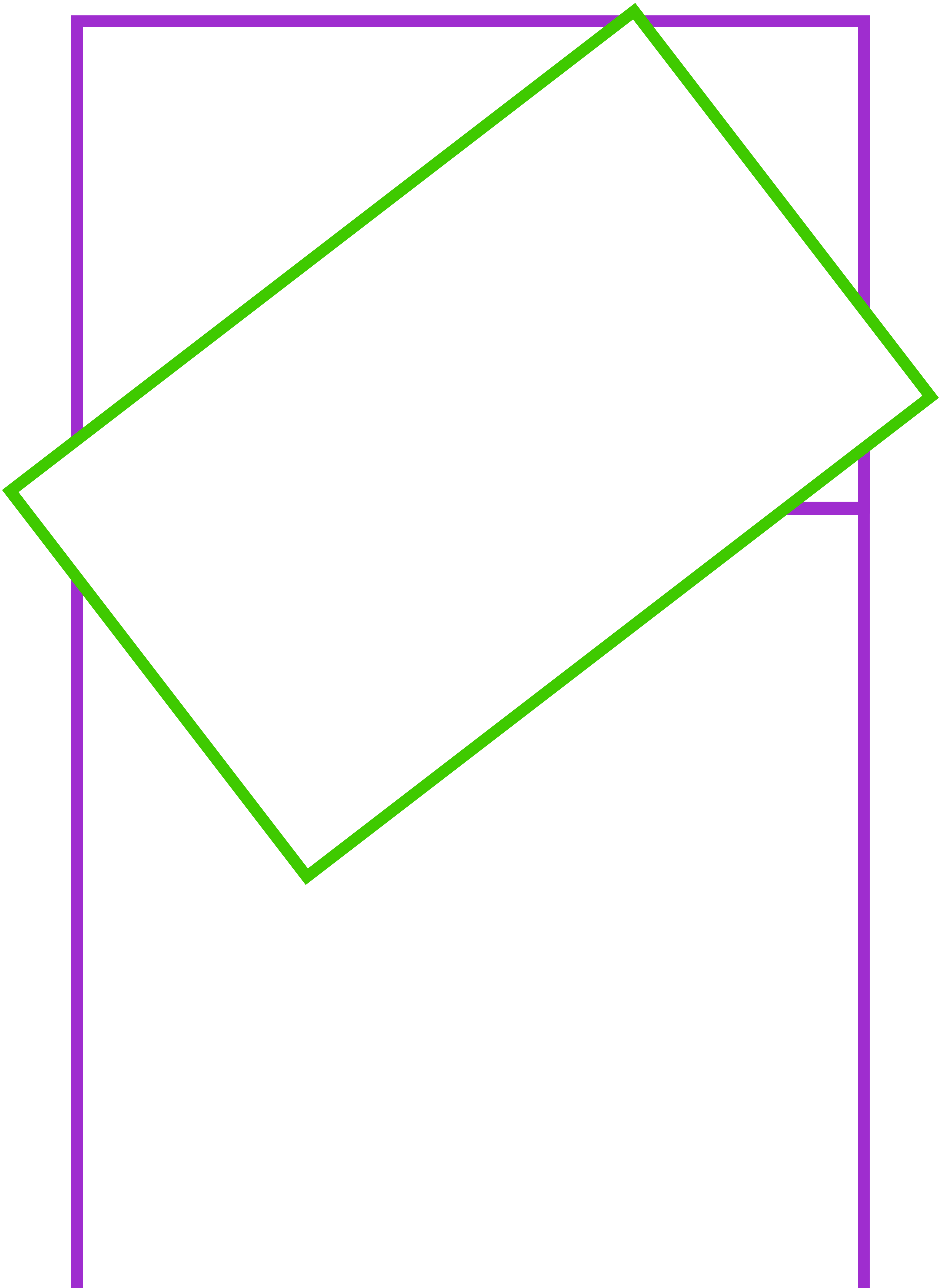


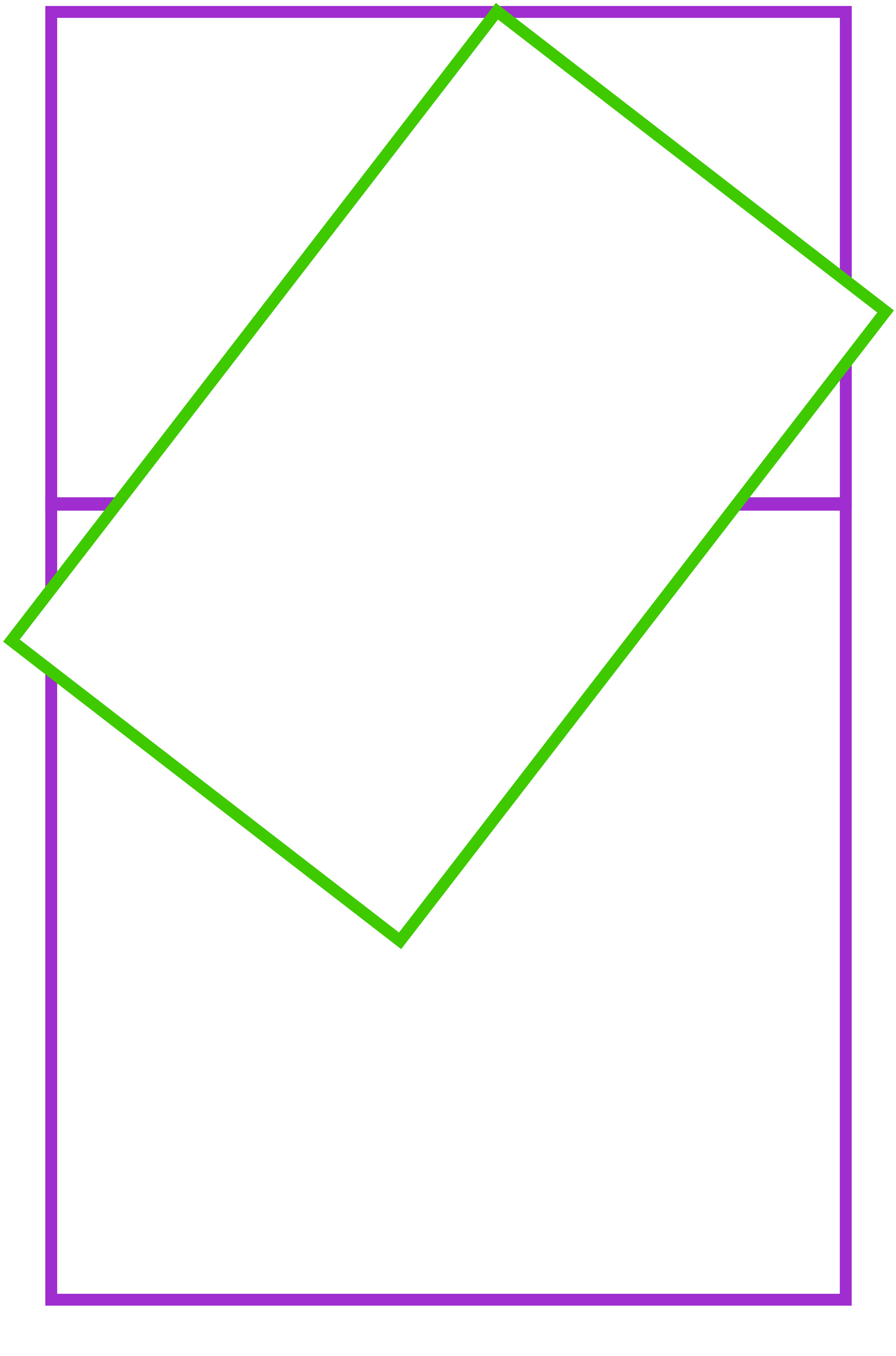


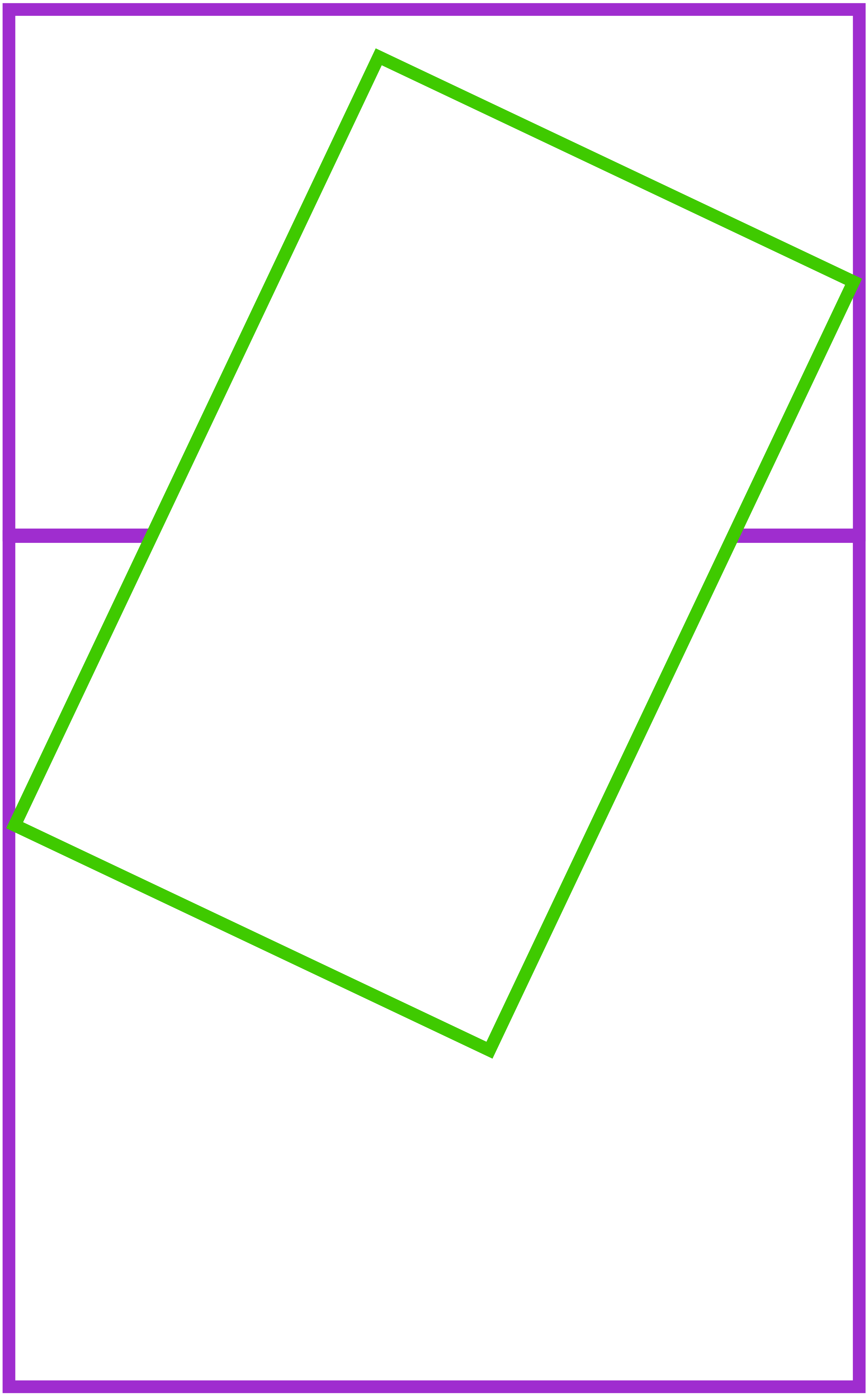


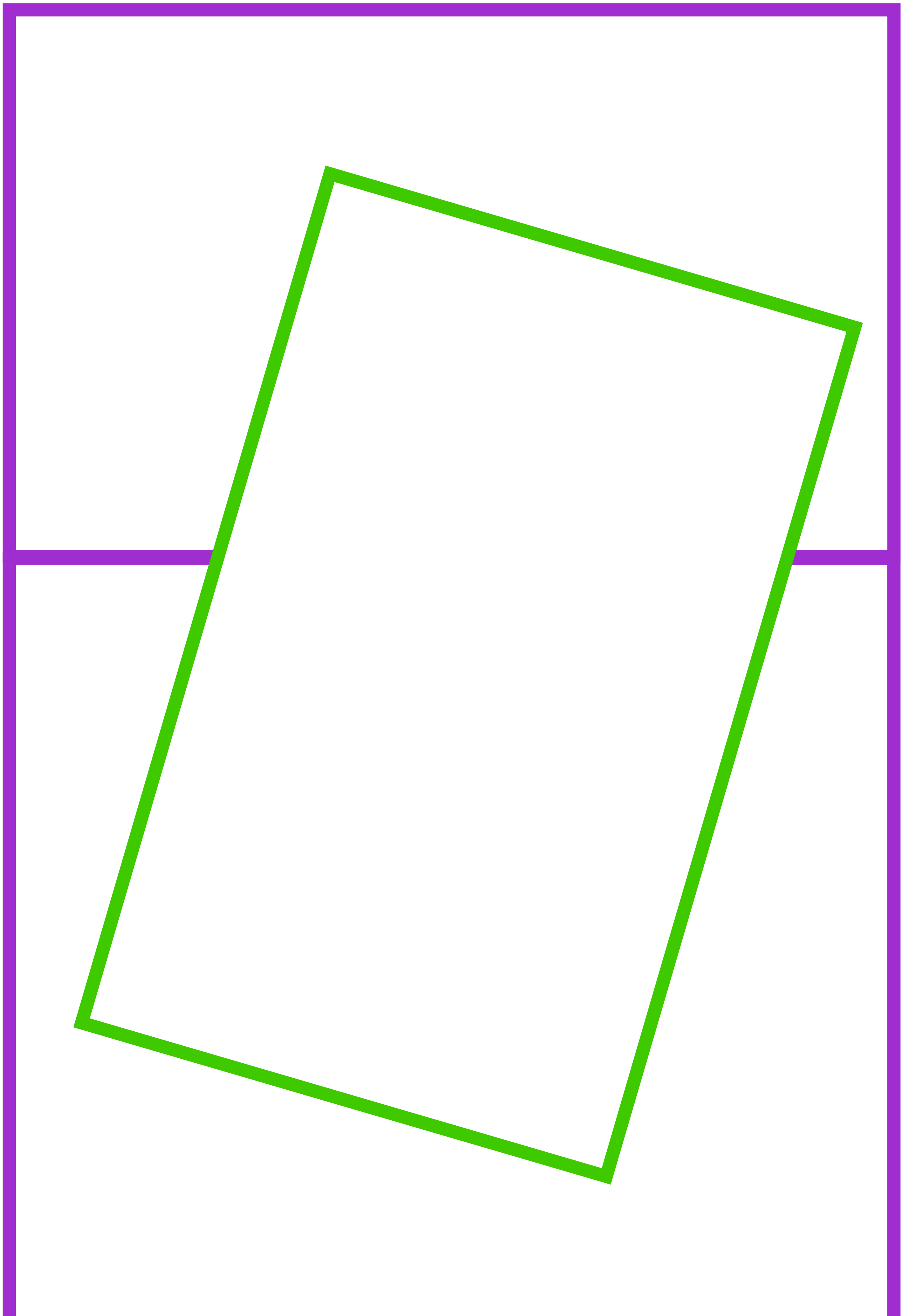


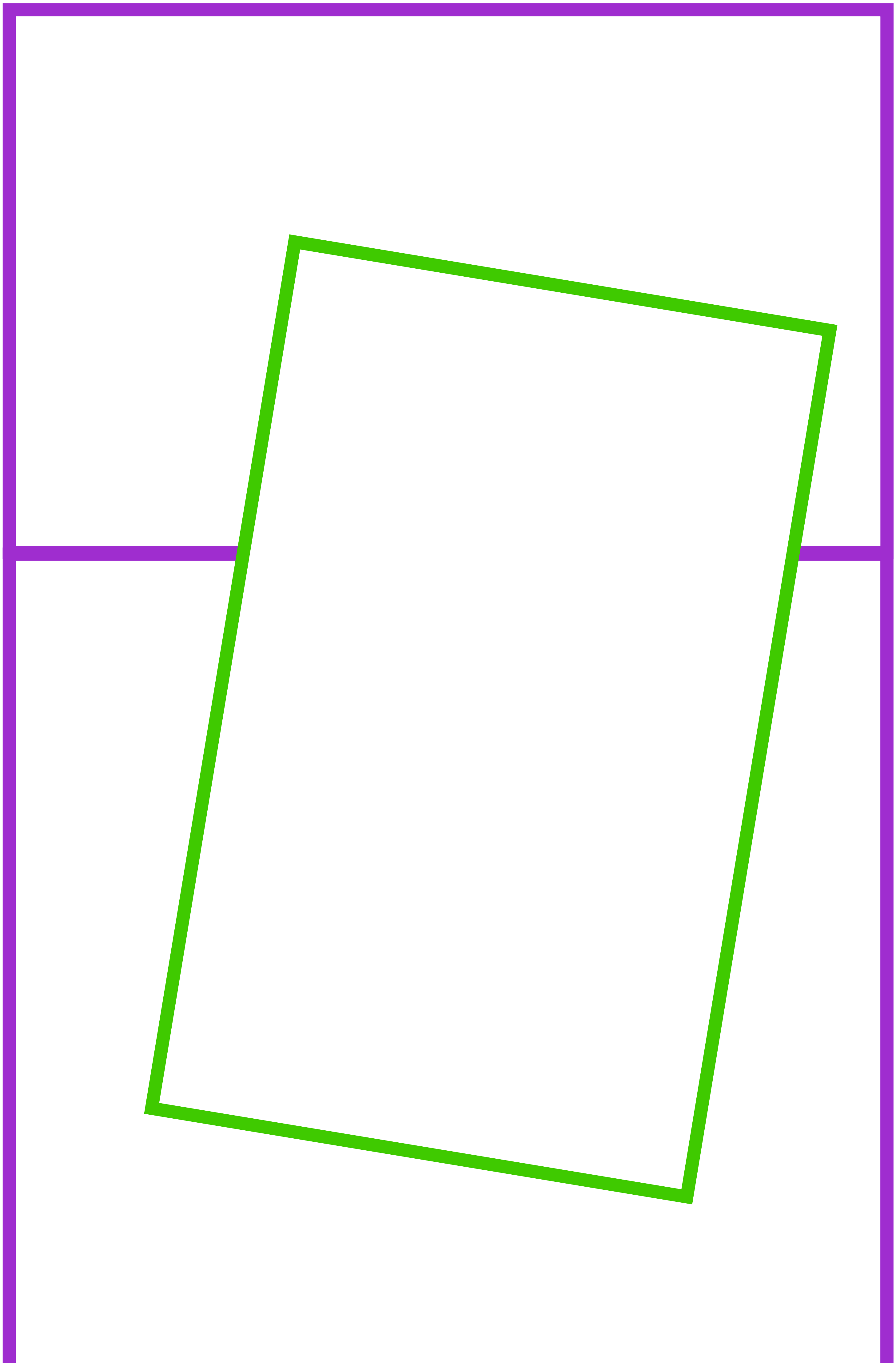


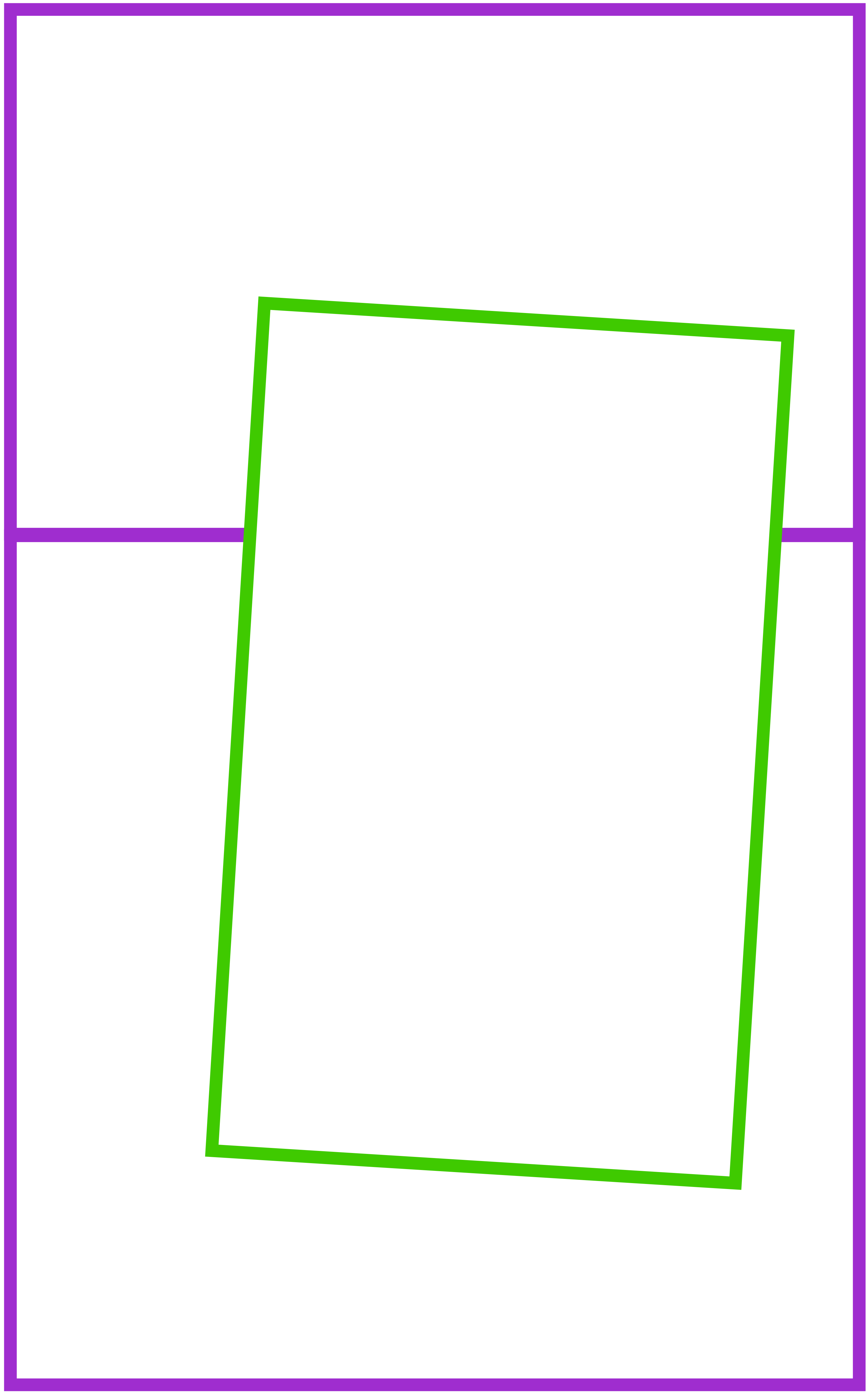


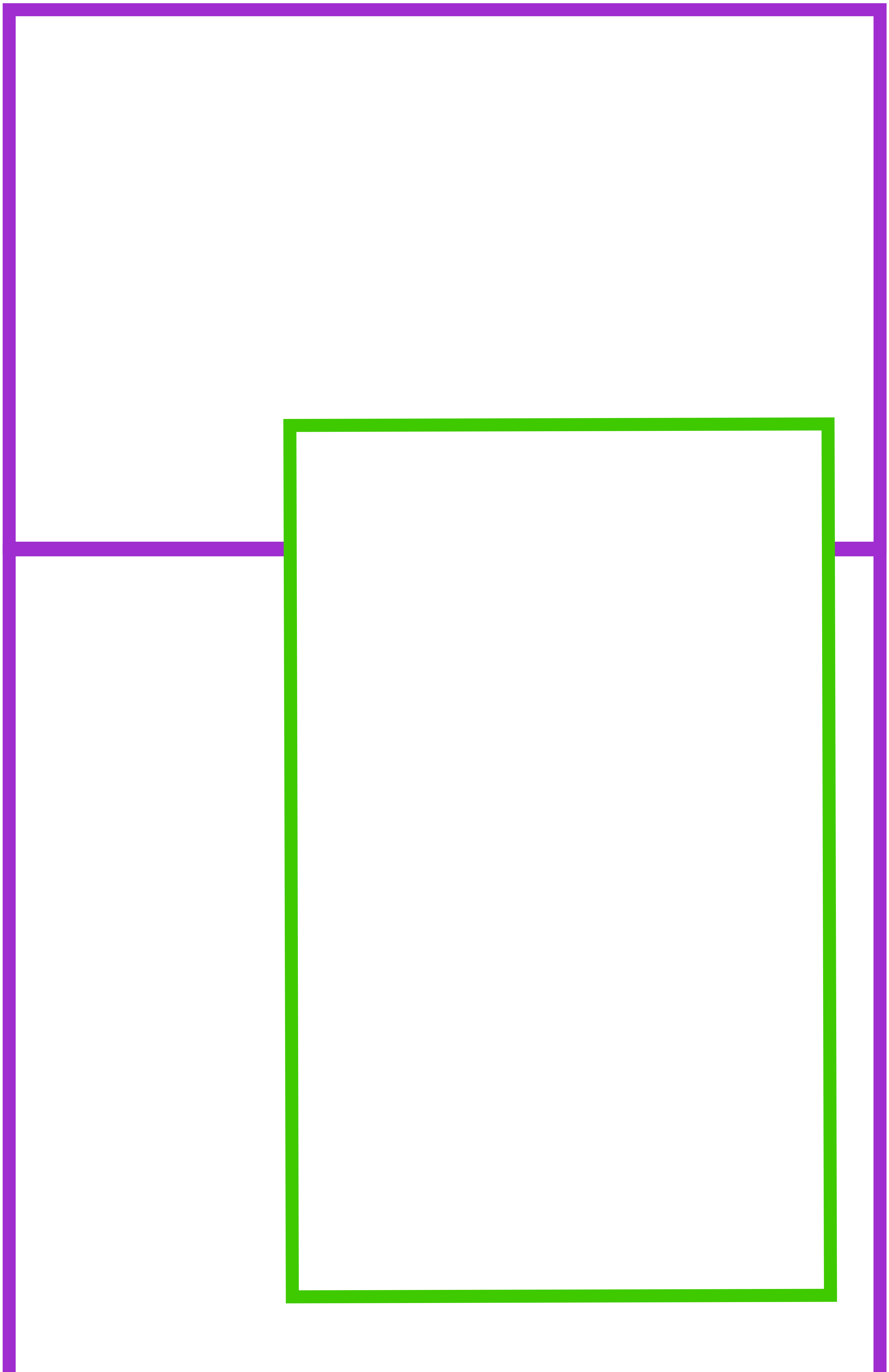


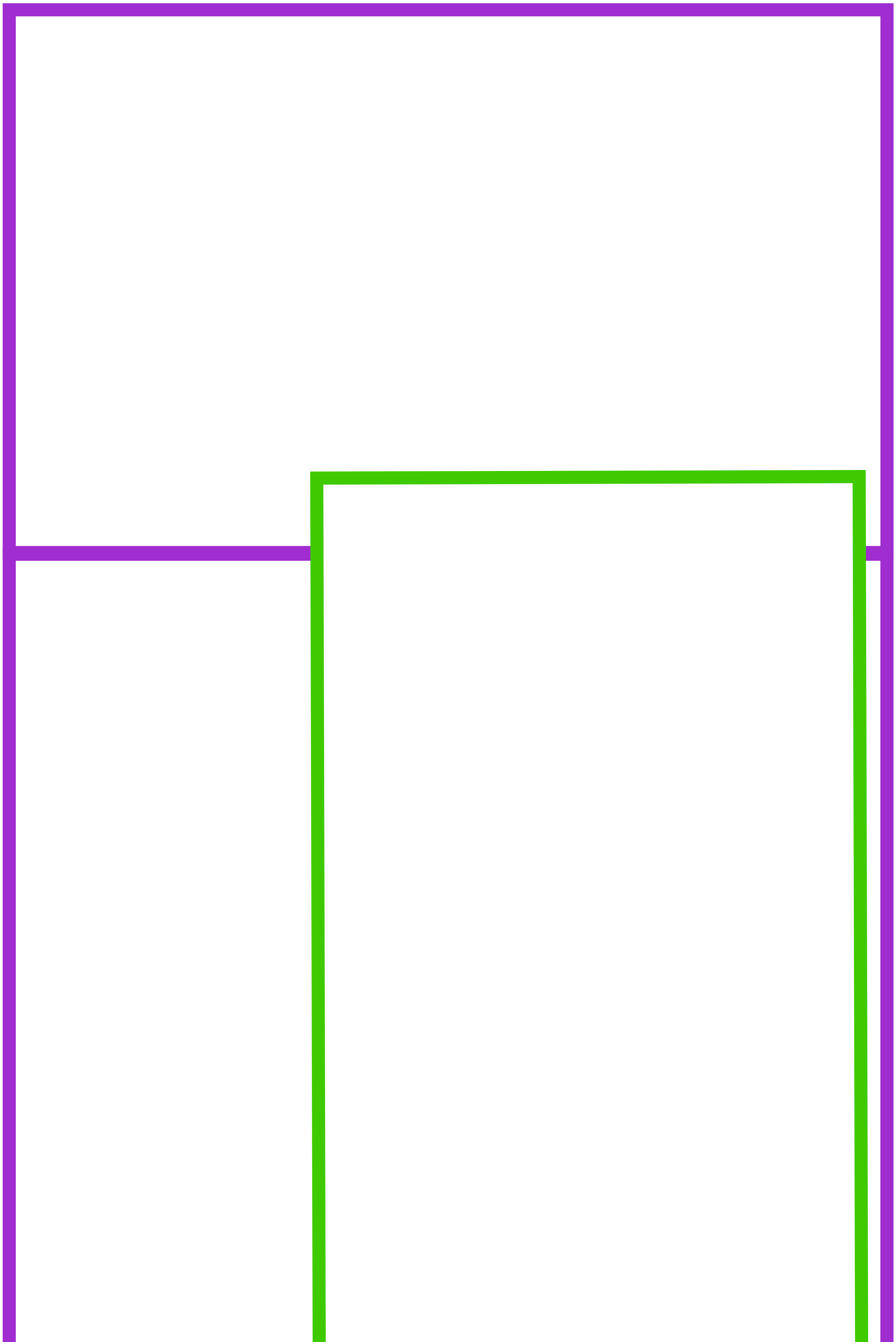


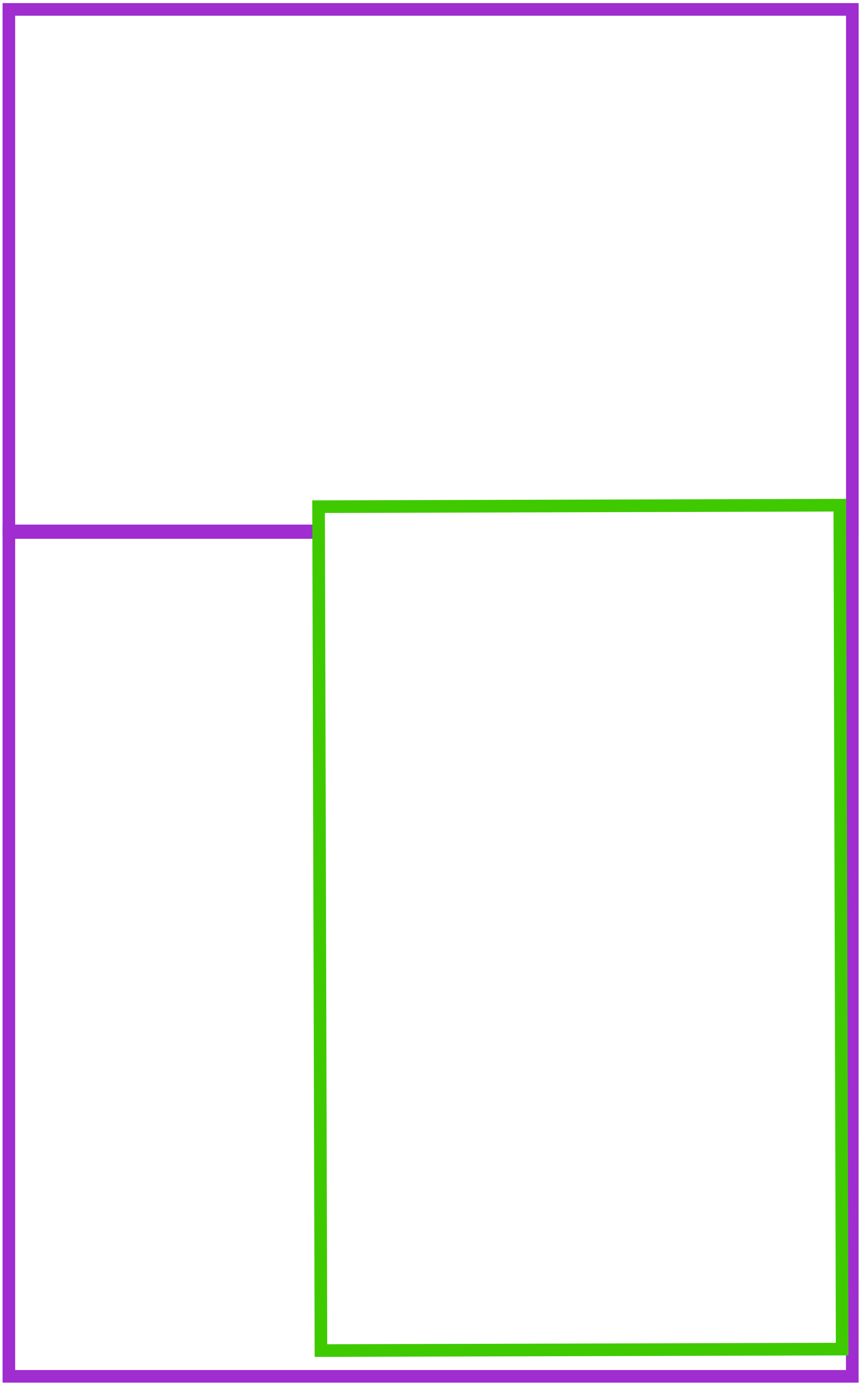


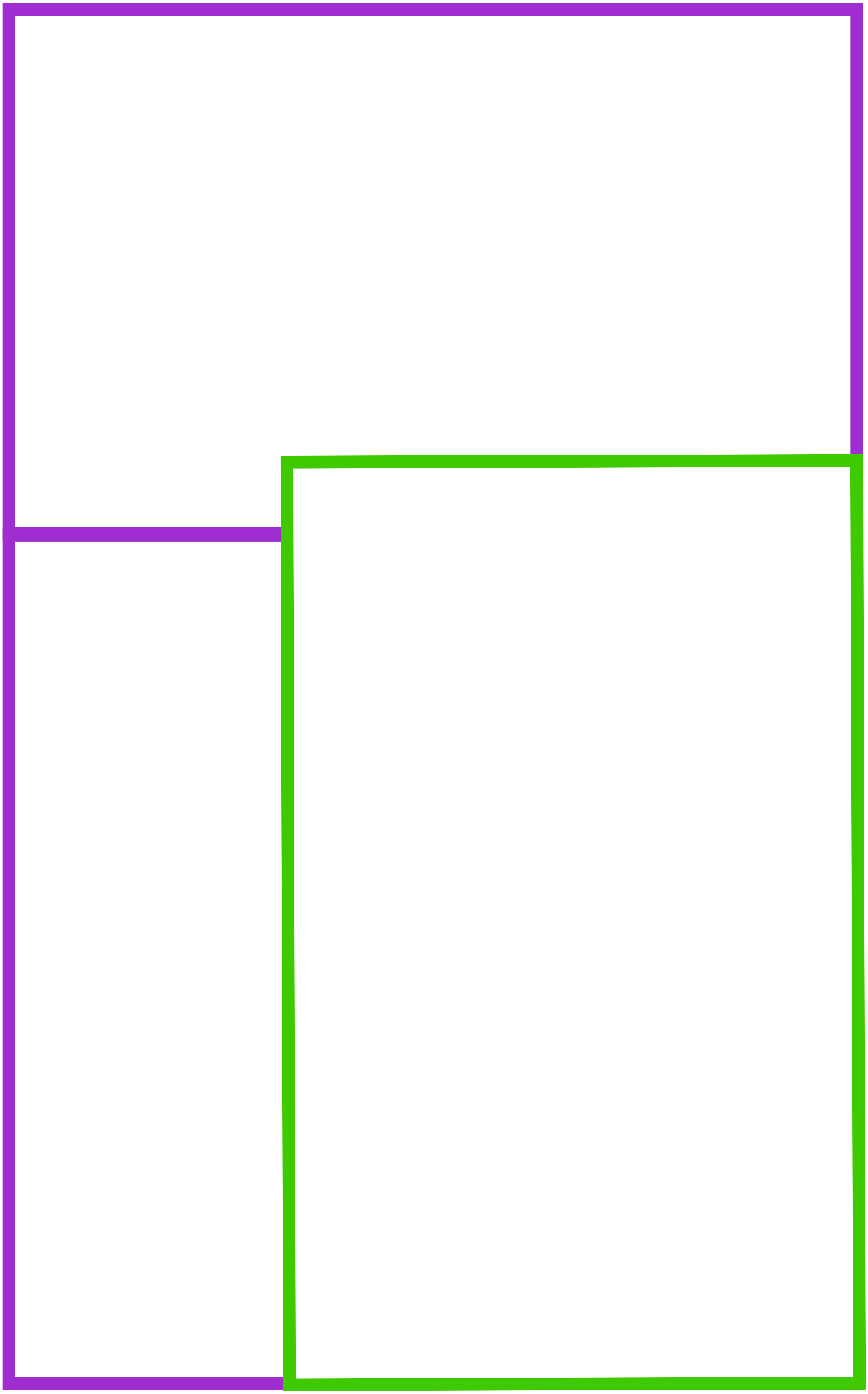


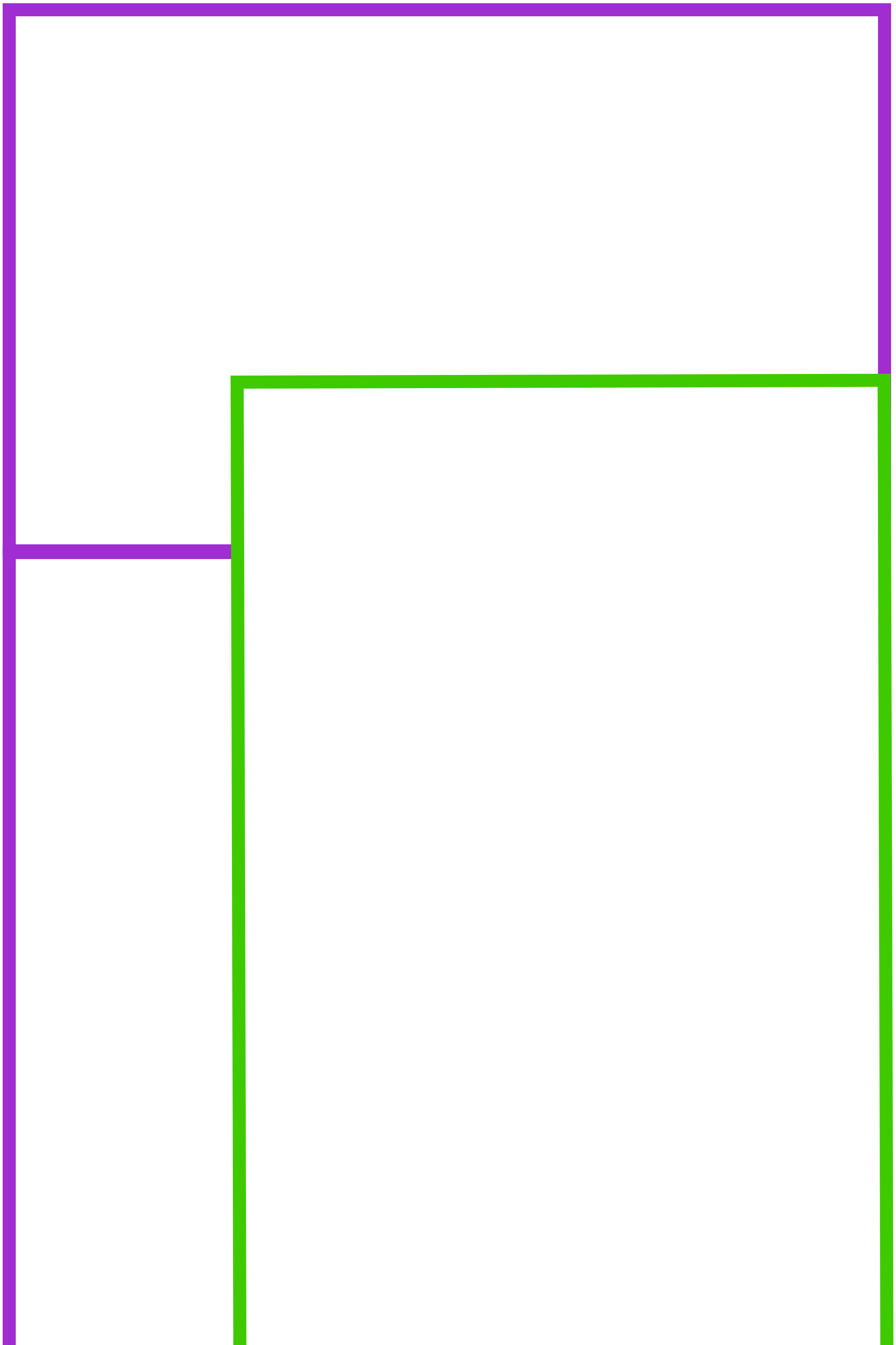


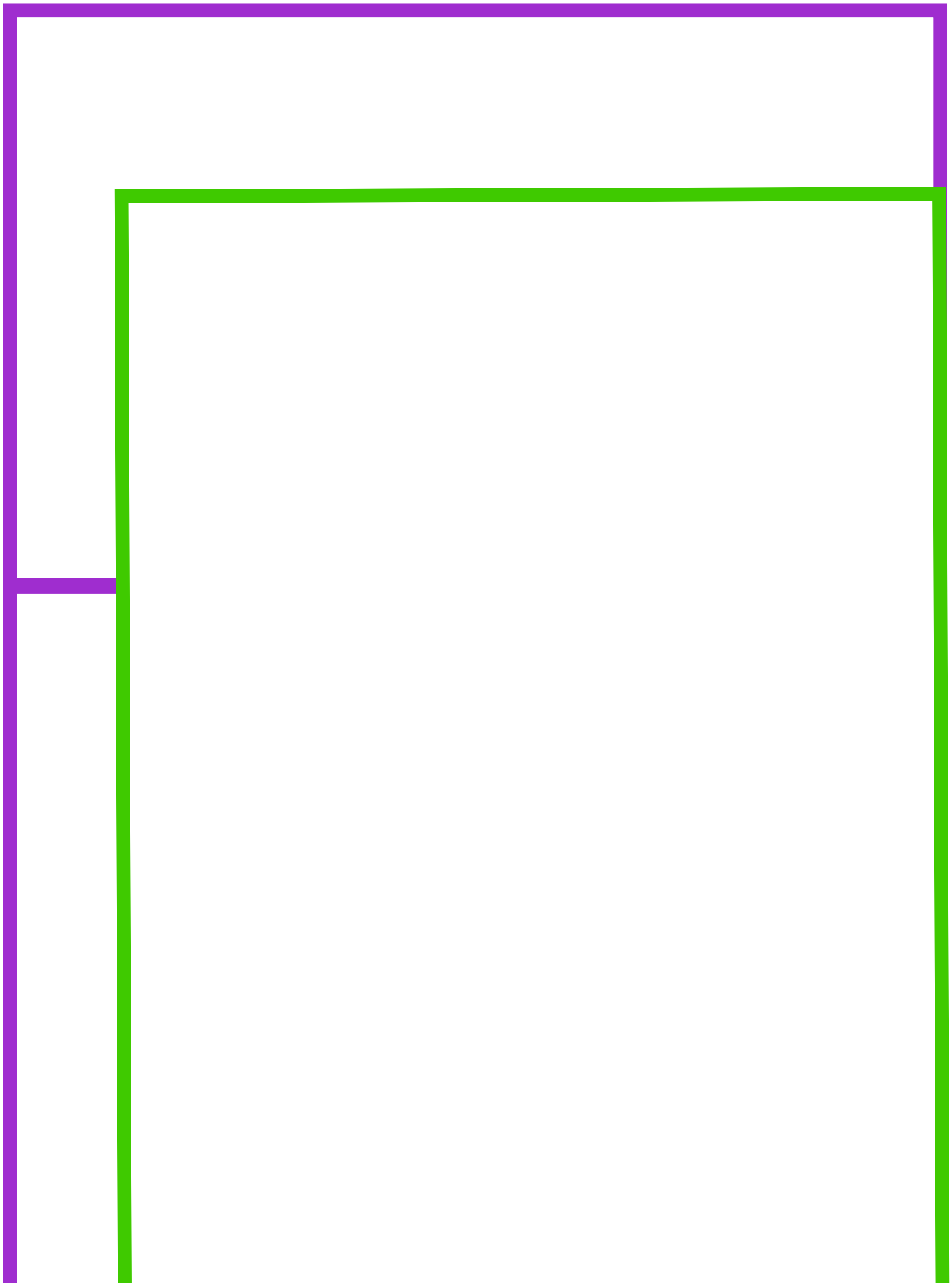


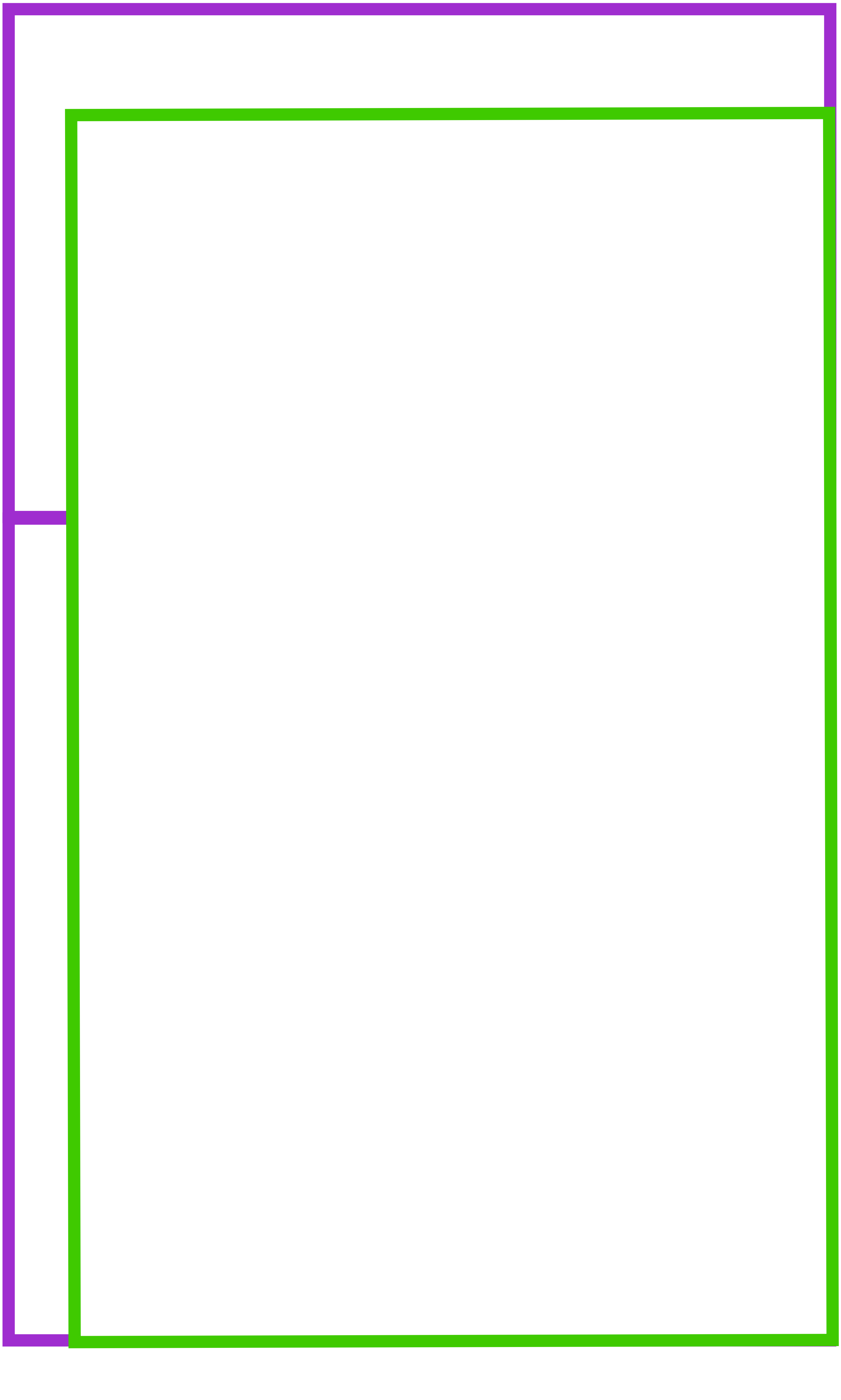


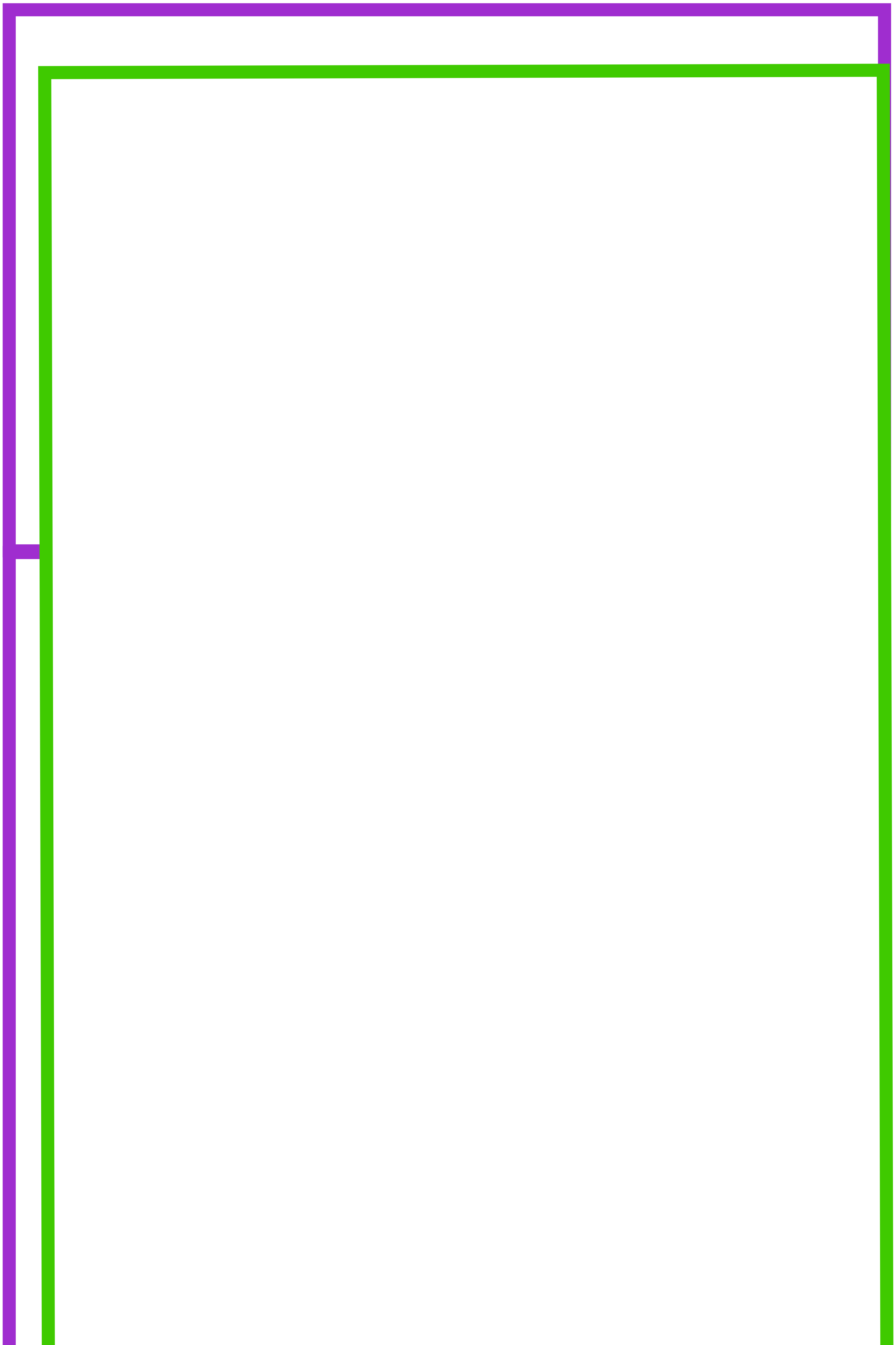


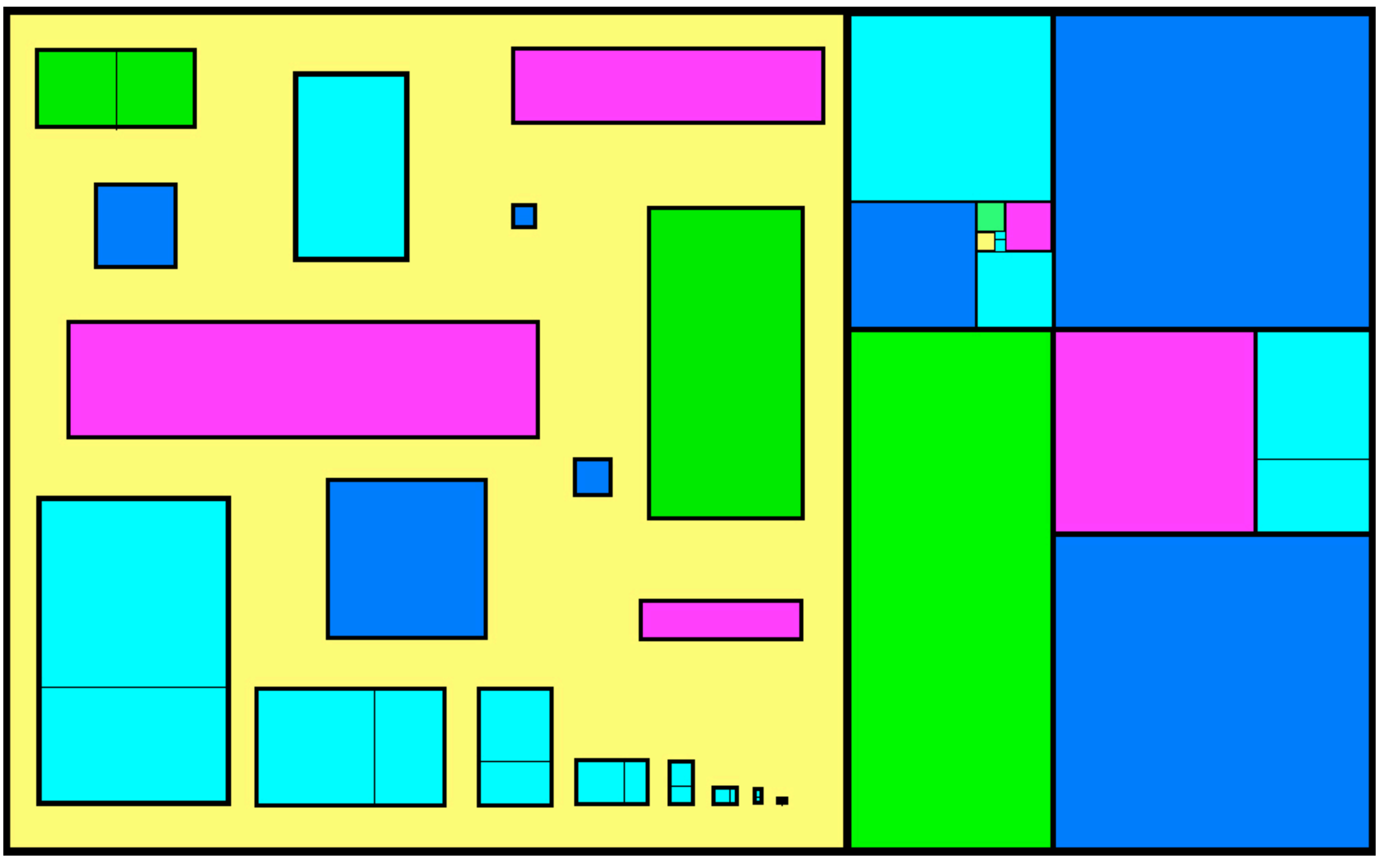


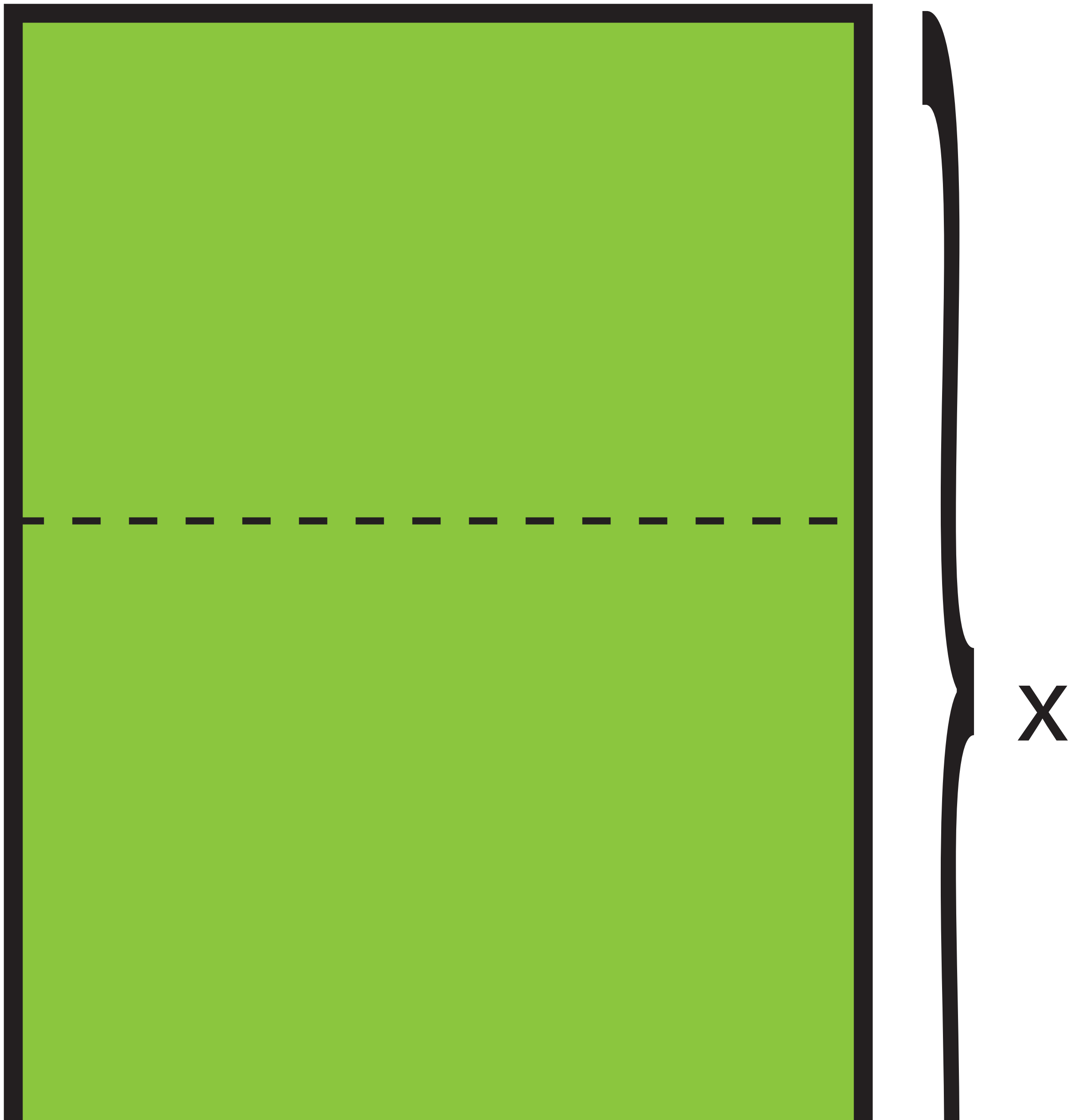











x
1

$$x/1 = 1/(x-1)$$

$$x(x-1) = 1$$

$$x^2 - x = 1$$

$$x^2 - x - 1 = 0$$

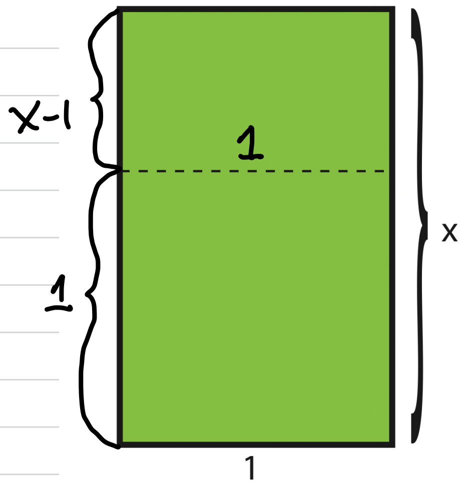
$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

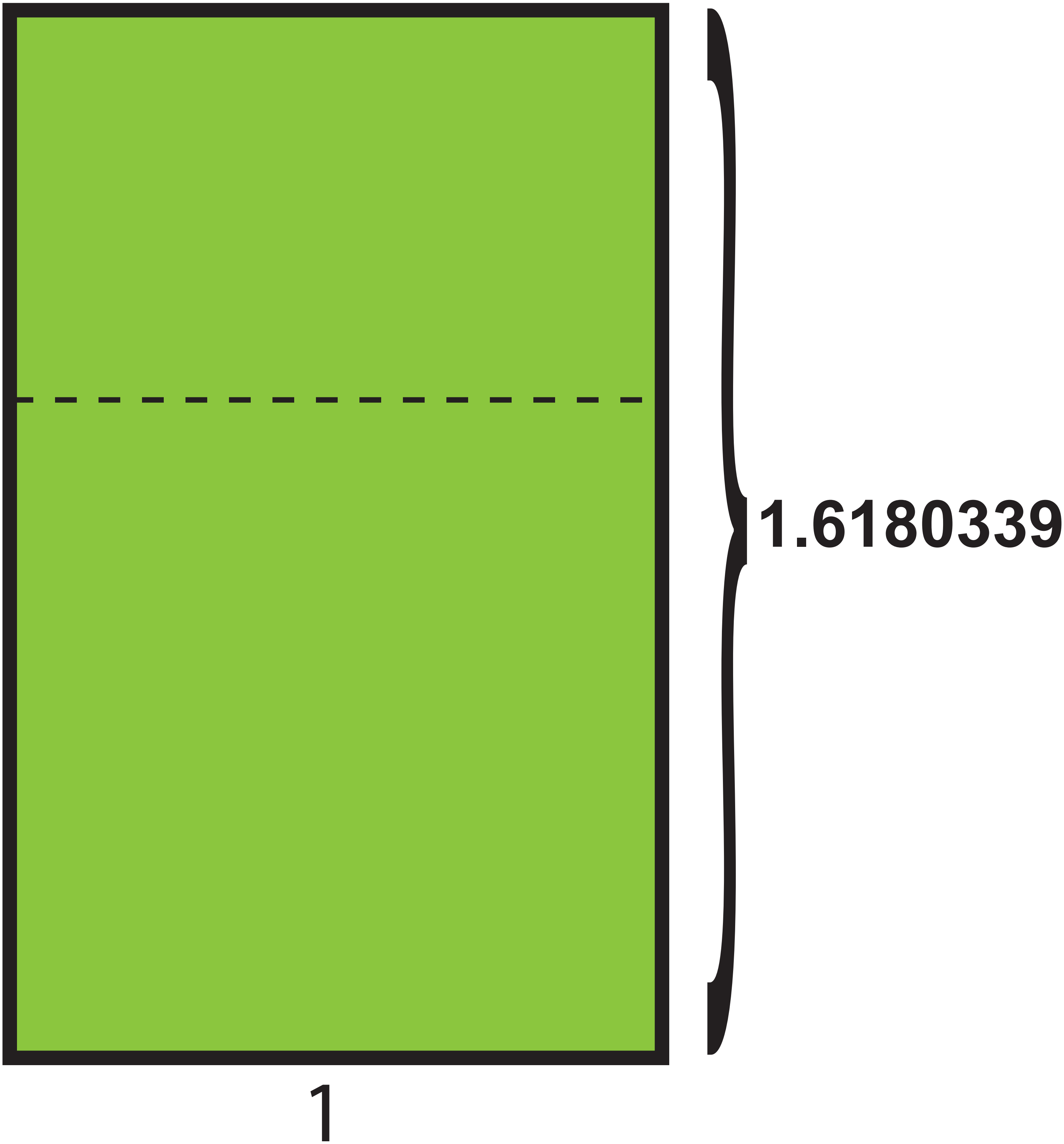
$$= \frac{1 \pm \sqrt{5}}{2}$$

taking the "–" sign gives a negative #. So take the +:

$$x = \frac{1 + \sqrt{5}}{2}$$

$$= 1.6180339$$

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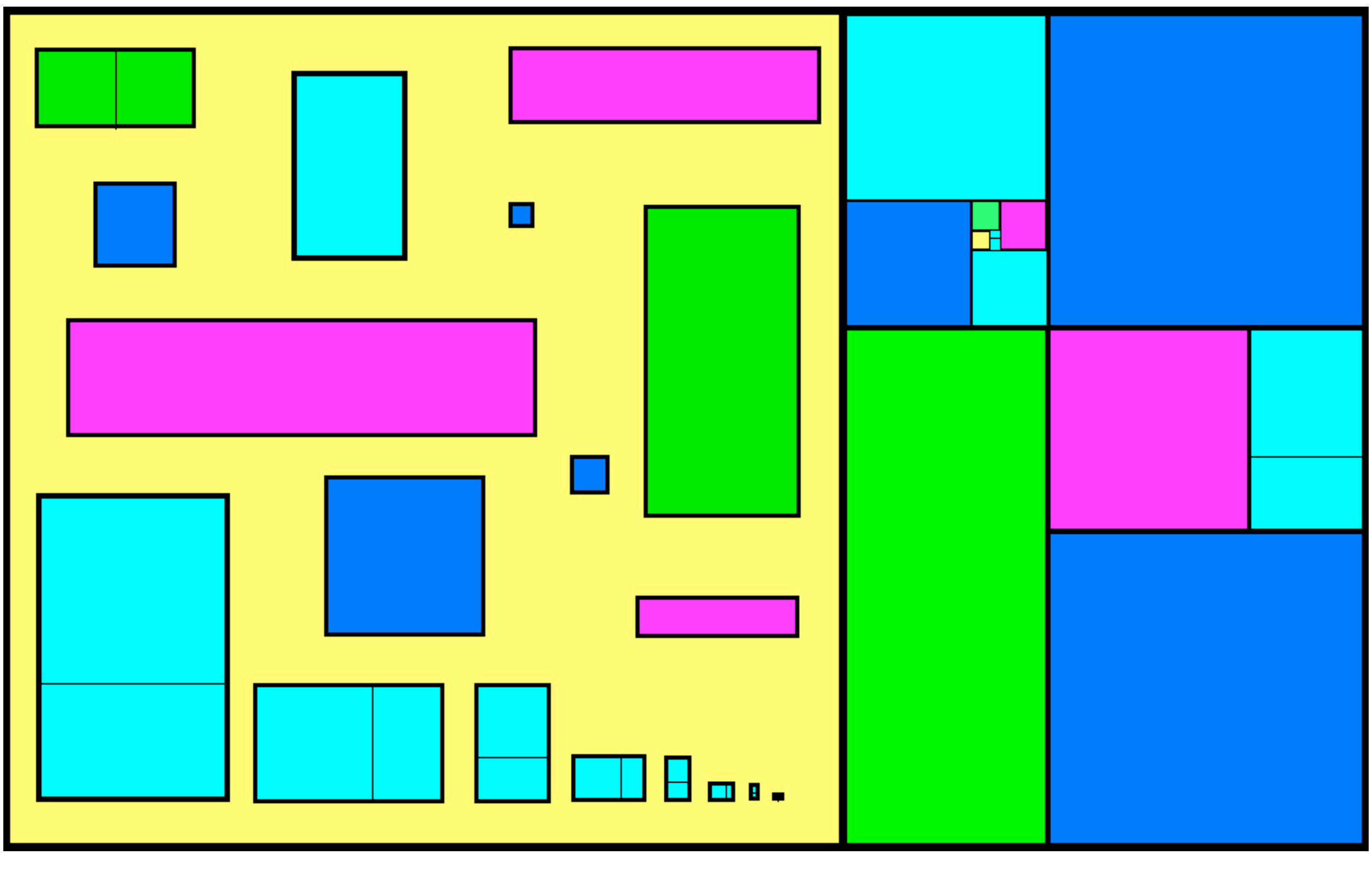
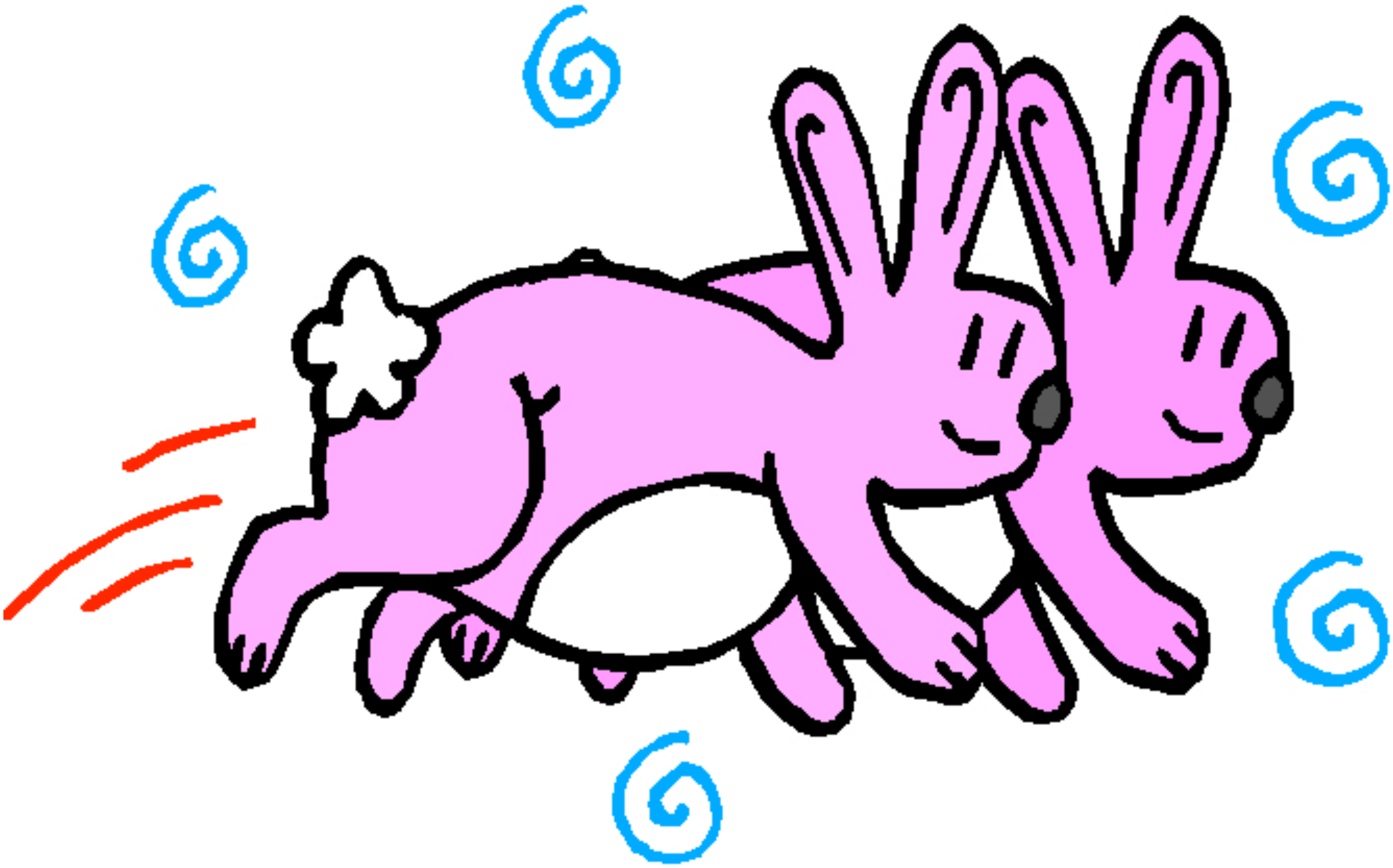
890244970720720
418939113748475
408807538689175

Some call it

Φ





Fibonacci Rabbits

In his 1202 book *Liber Abaci*, Leonardo of Pisa (also called "Fibonacci," meaning "son of the good-natured one") considered the growth of a population of PAIRS of rabbits, subject to the following rules.

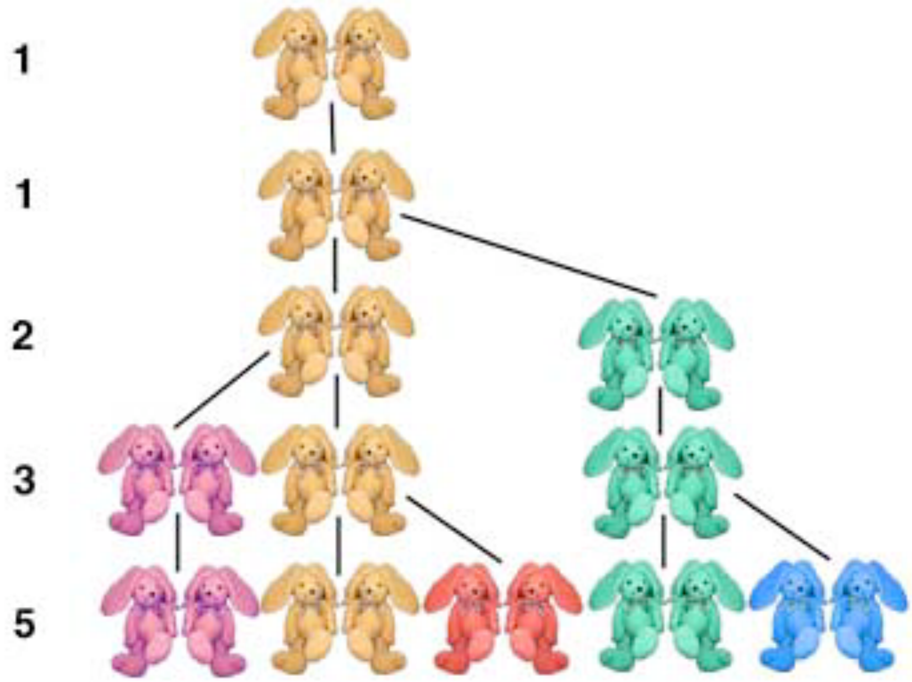
RULES FOR RABBIT REPRODUCTION

- **We begin with just 1 pair of newly born ("zero-month-old") rabbits (1 male, 1 female).**
- **Beginning in the SECOND month of life, a pair of rabbits will produce another pair of newly born rabbits every month.**
- **Rabbits never die.**
- **Whenever a new pair of rabbits is produced, it is always a male and a female.**

FIBONACCI'S QUESTION: How many PAIRS of rabbits are there in any given month?

GROWN-UP RABBITS MULTIPLY,

BABY RABBITS GROW UP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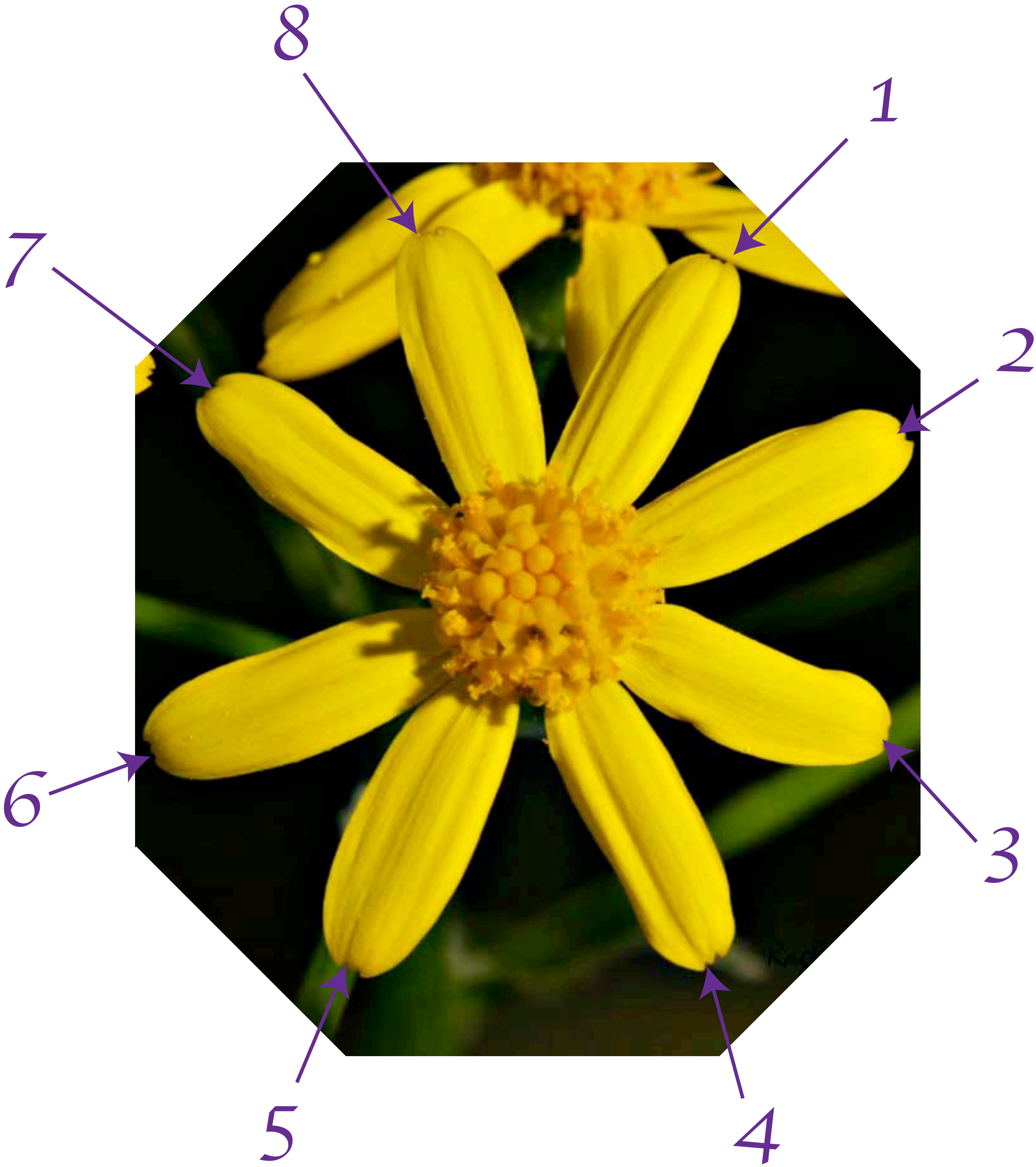
FIBONACCI'S QUESTION, ANSWERED!!!!

The number of PAIRS of rabbits grows, from month to month, like this:

1, 1, 2, 3, 5, 8, 13,
21, 34, 55, 89, 144, ...

These numbers are called the Fibonacci numbers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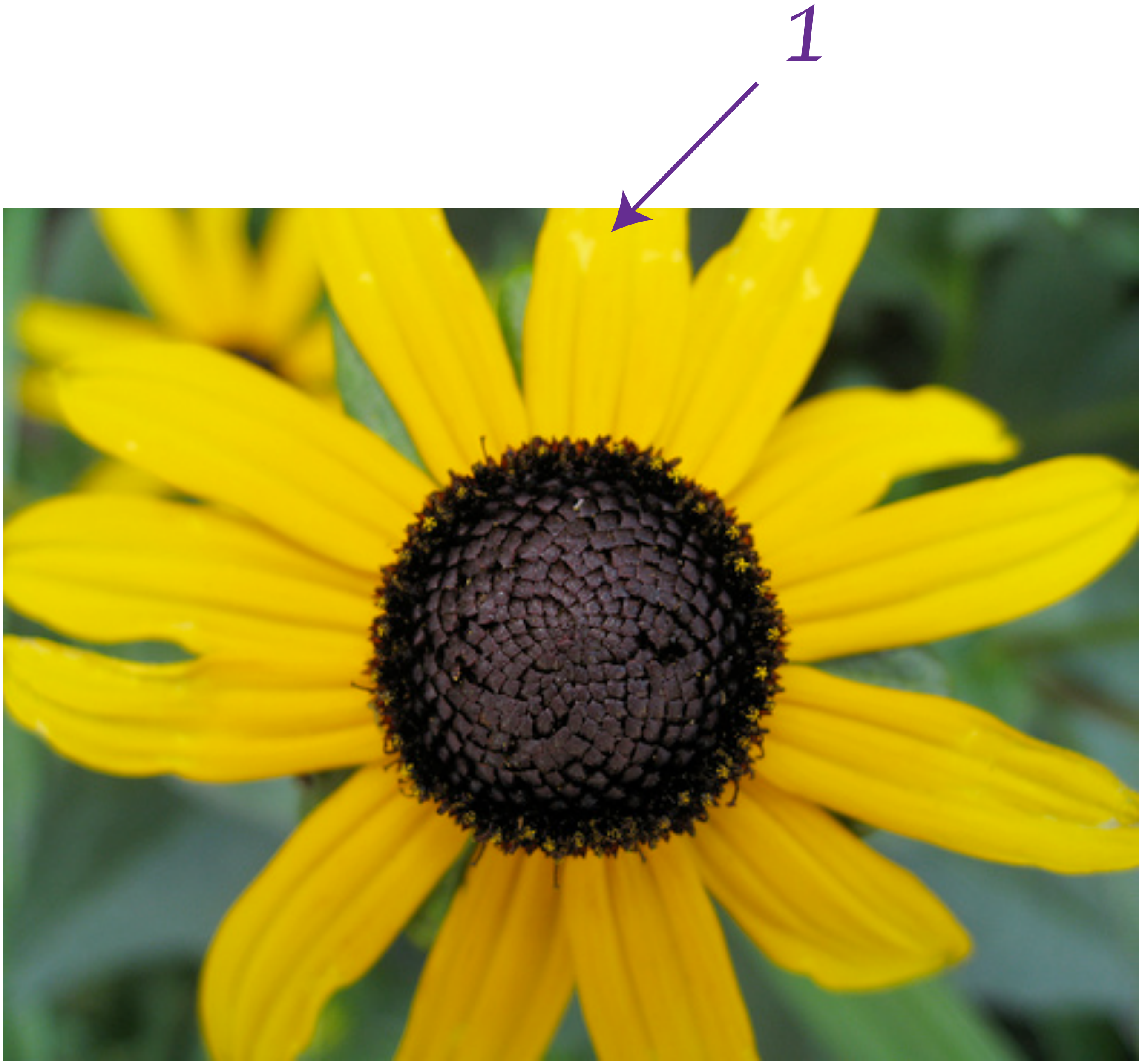
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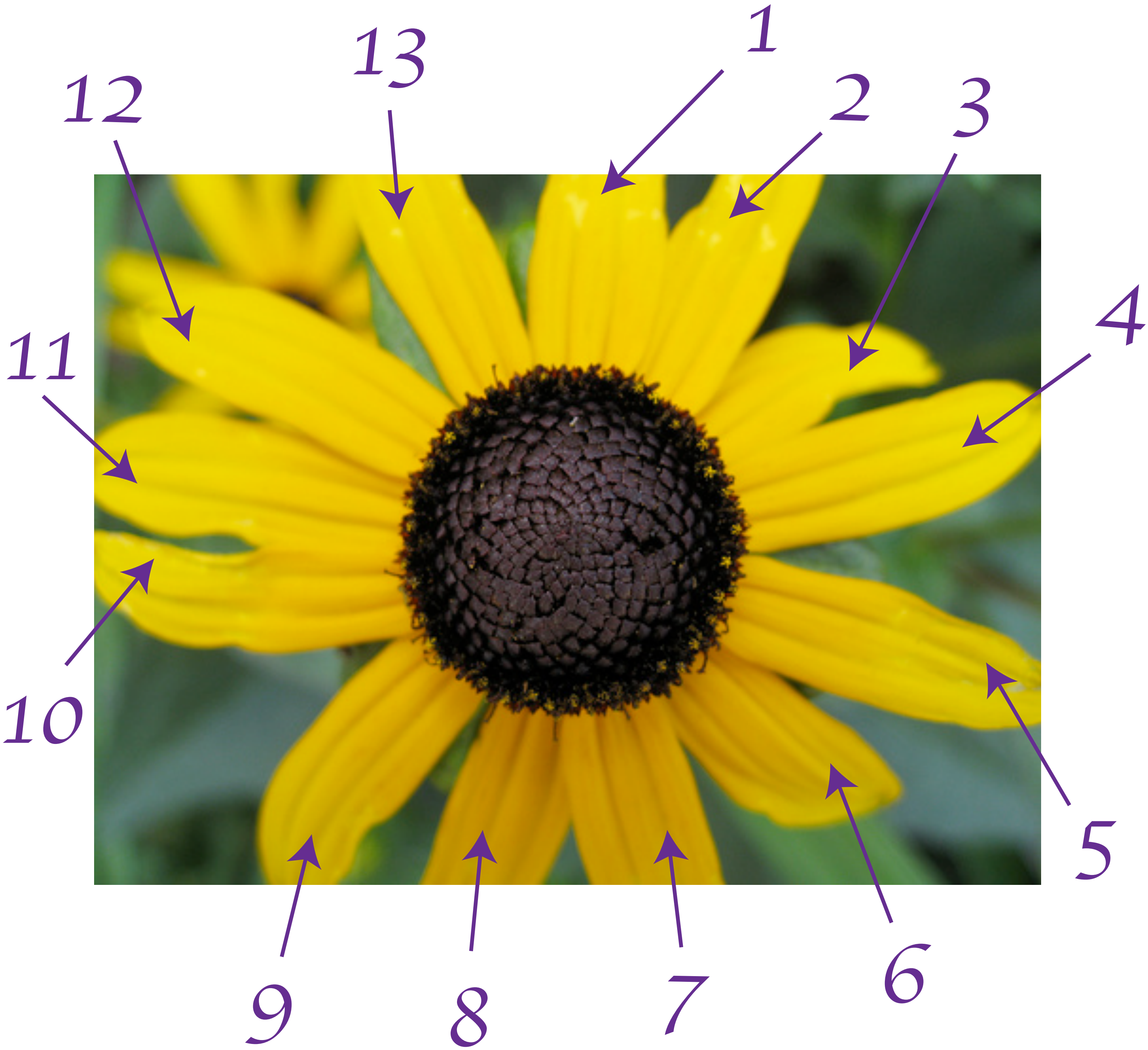
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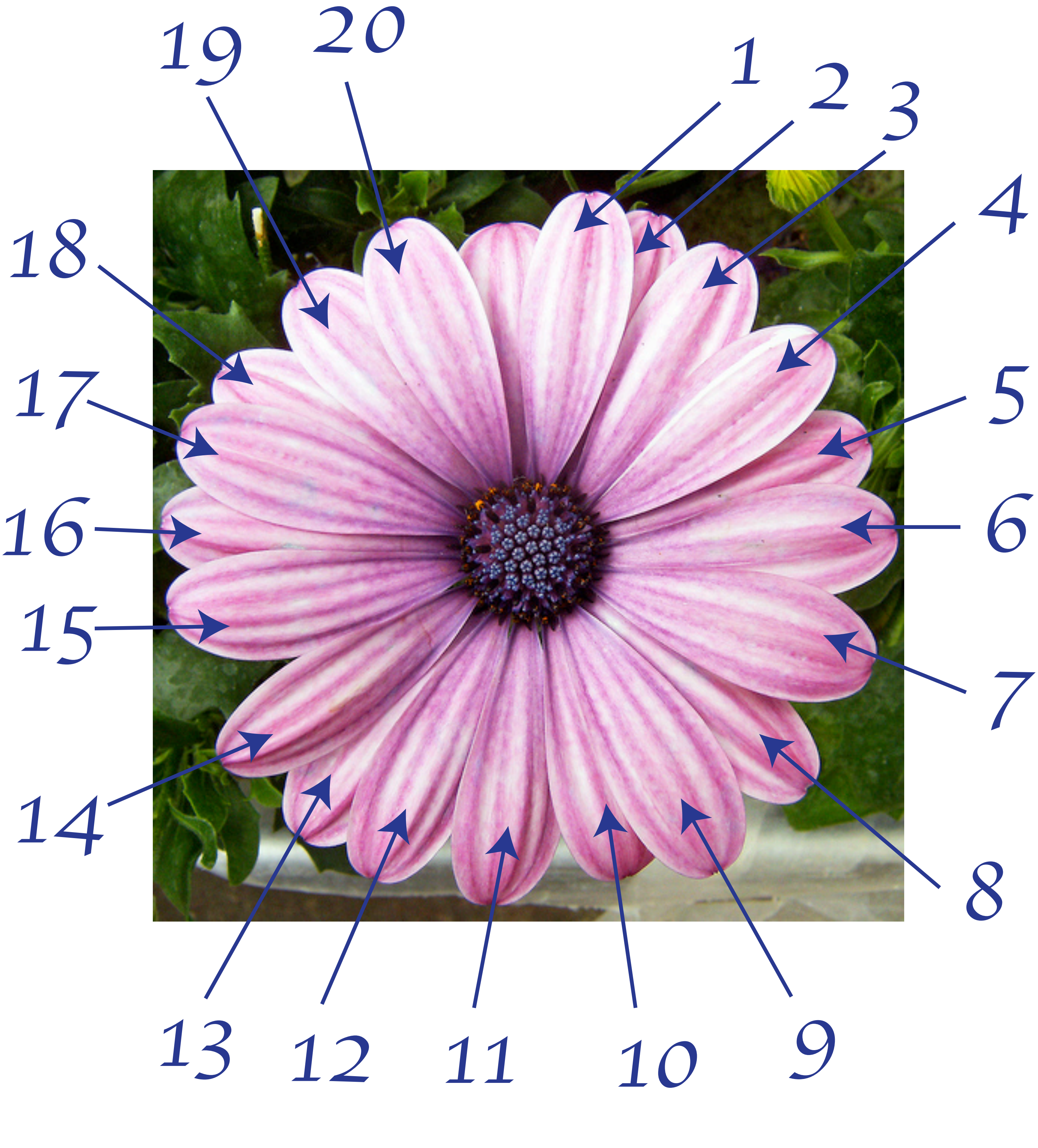
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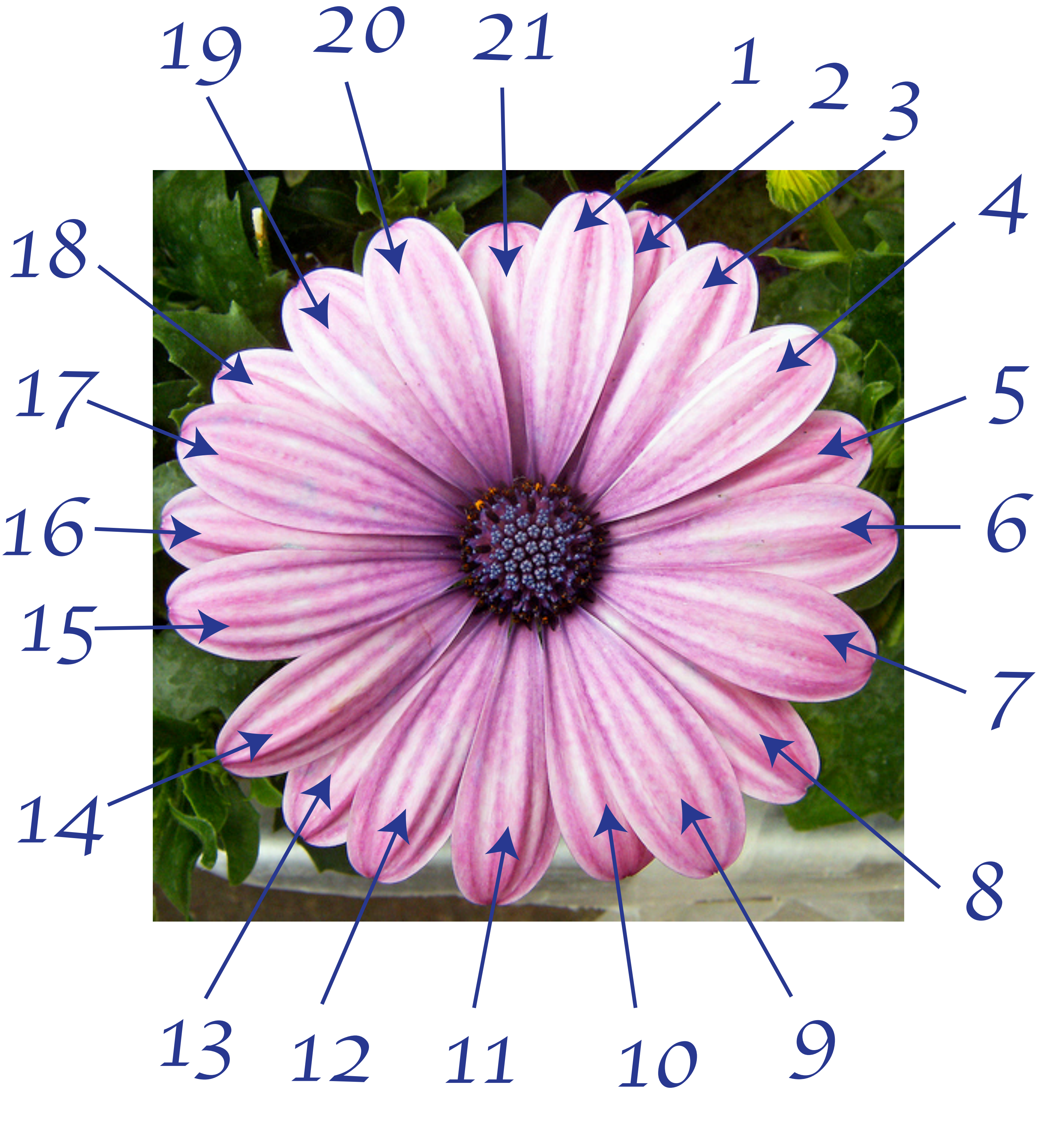
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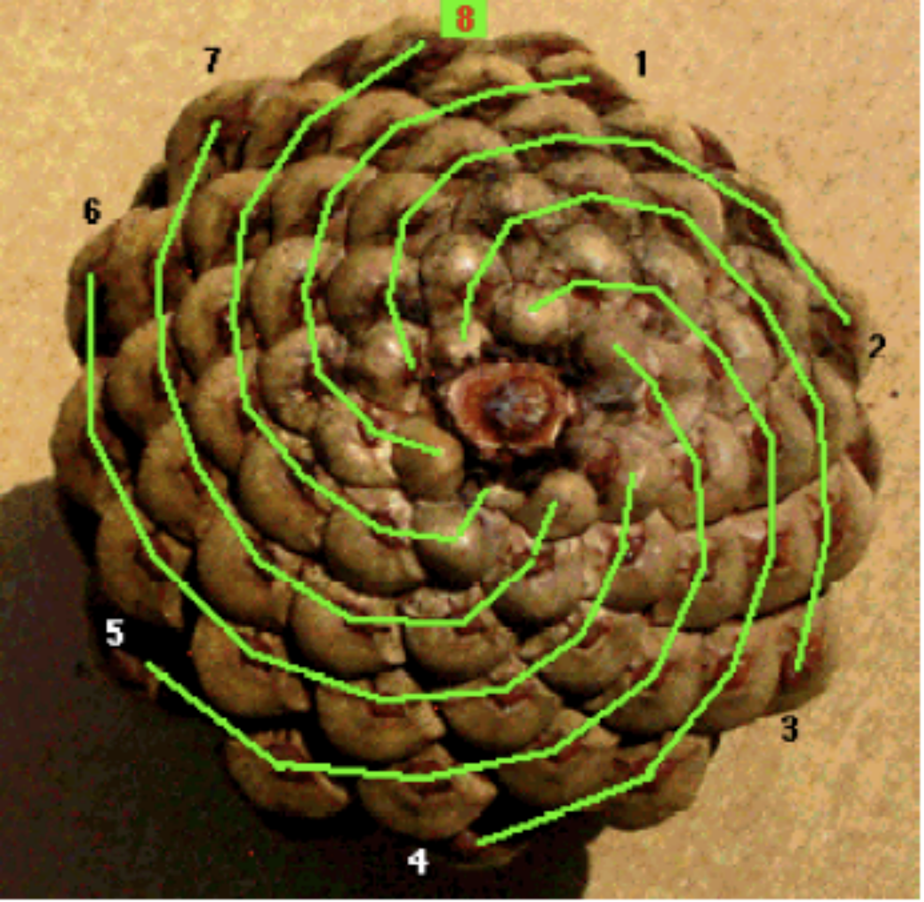
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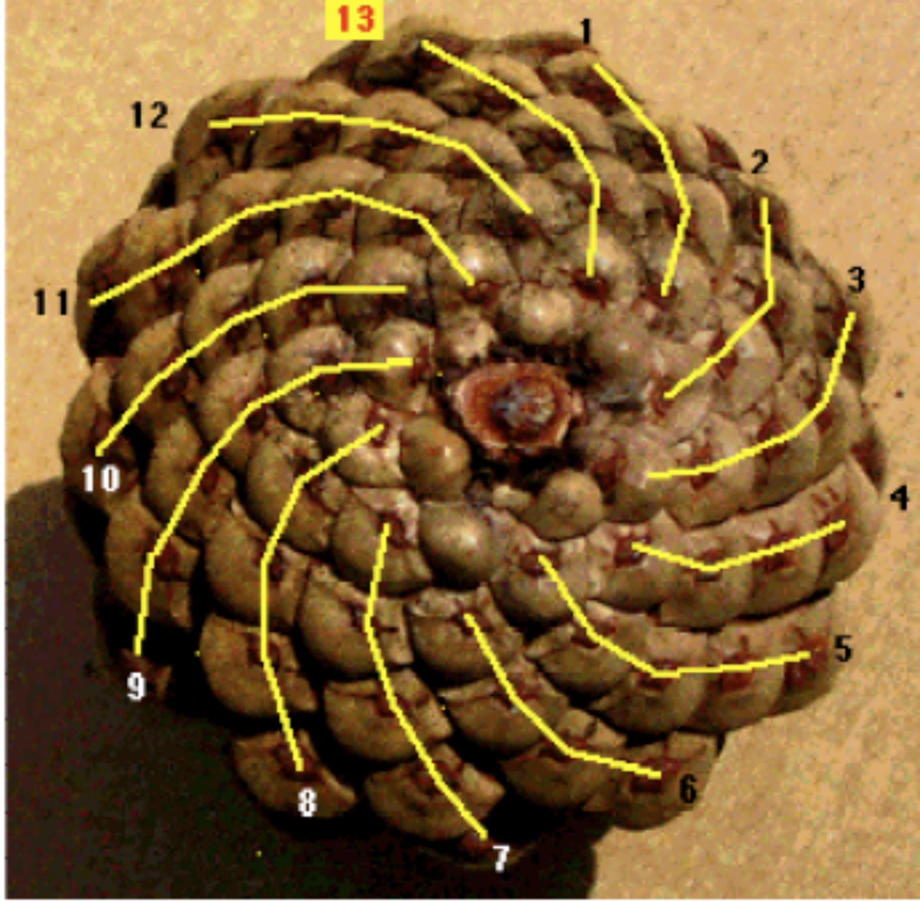
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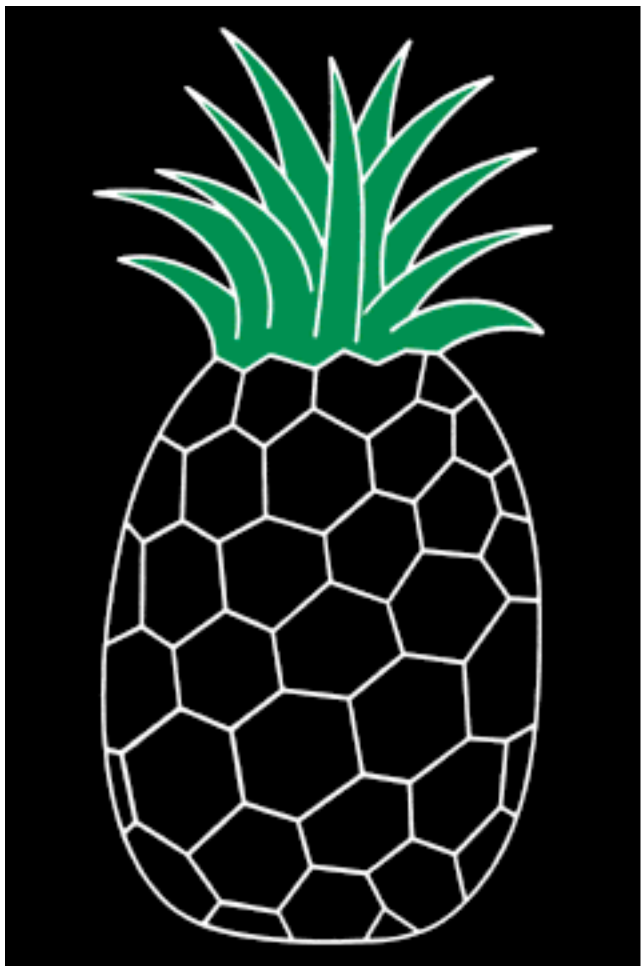

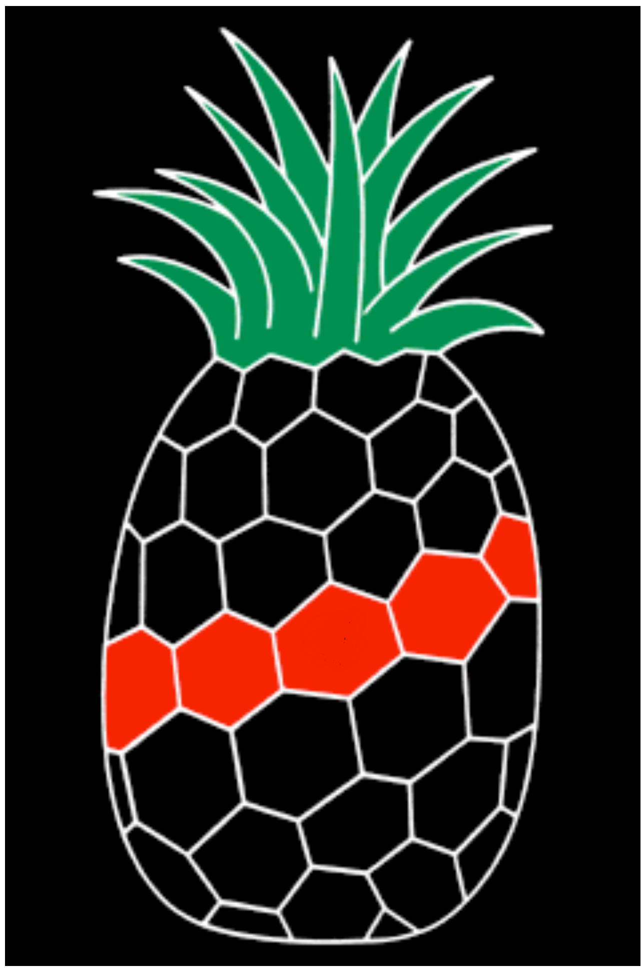

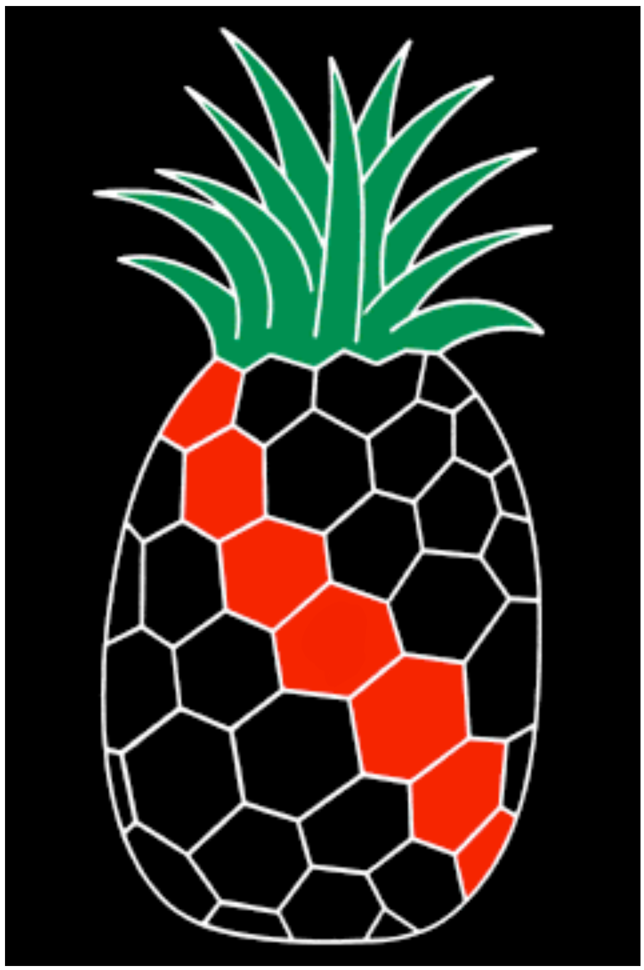

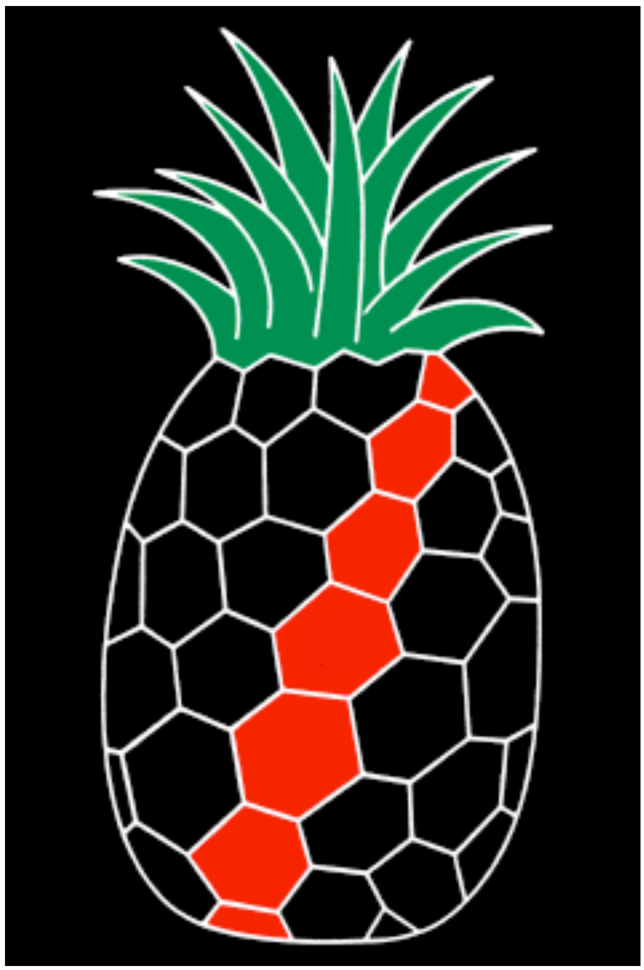

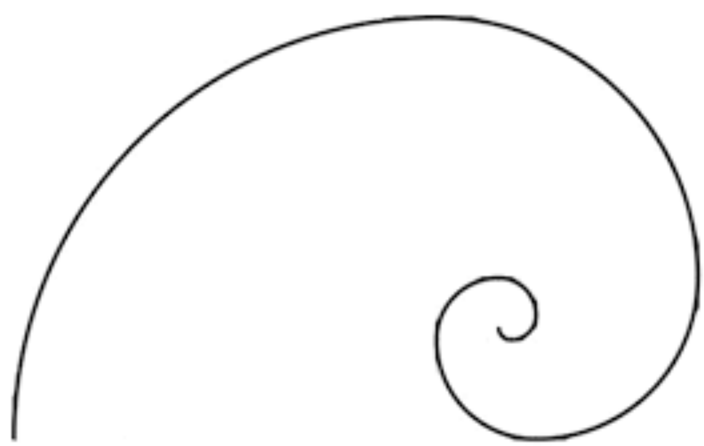

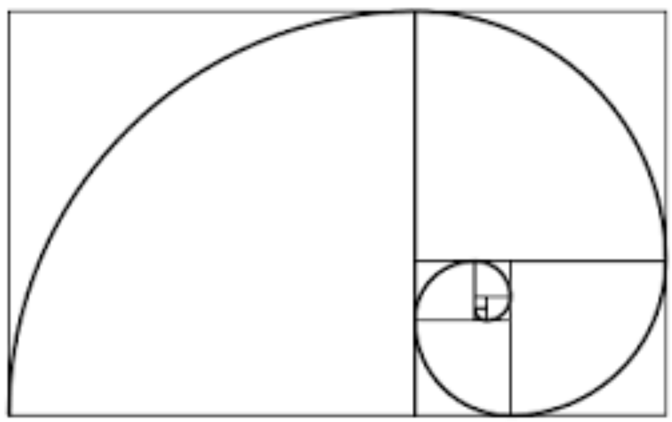

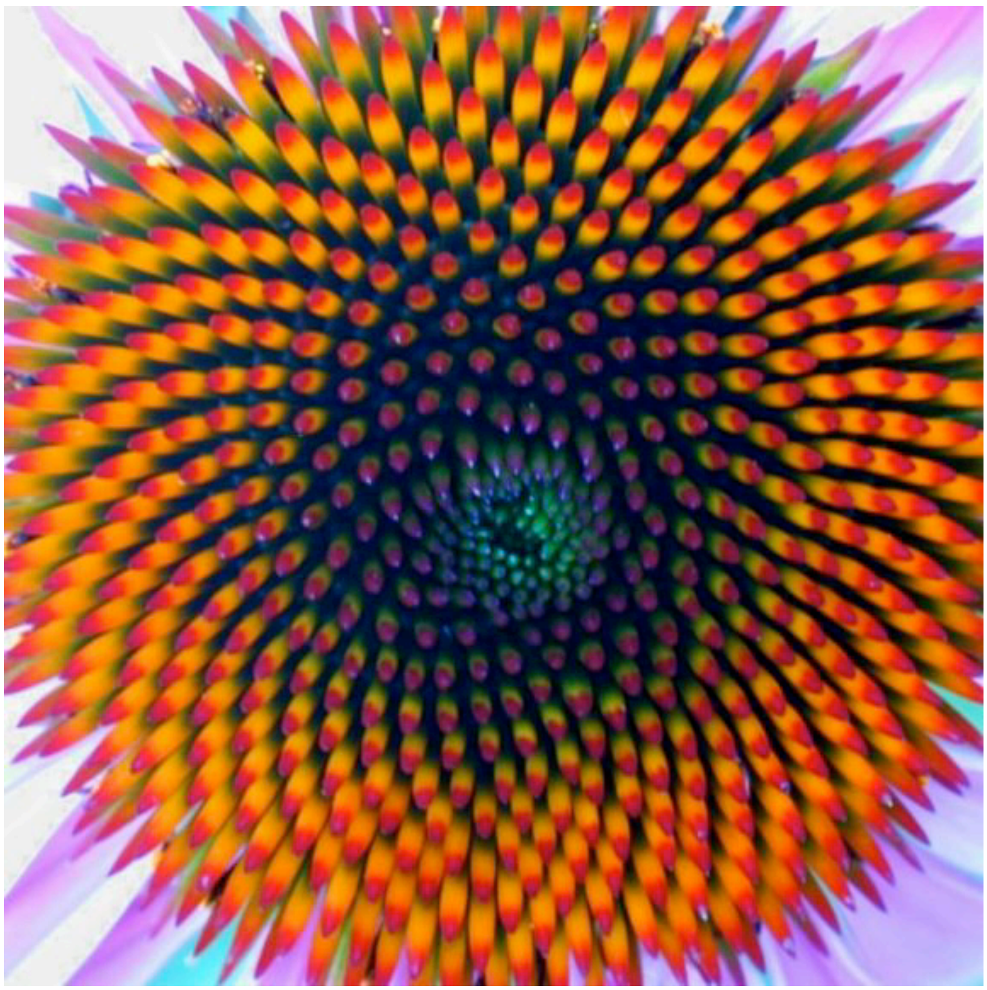

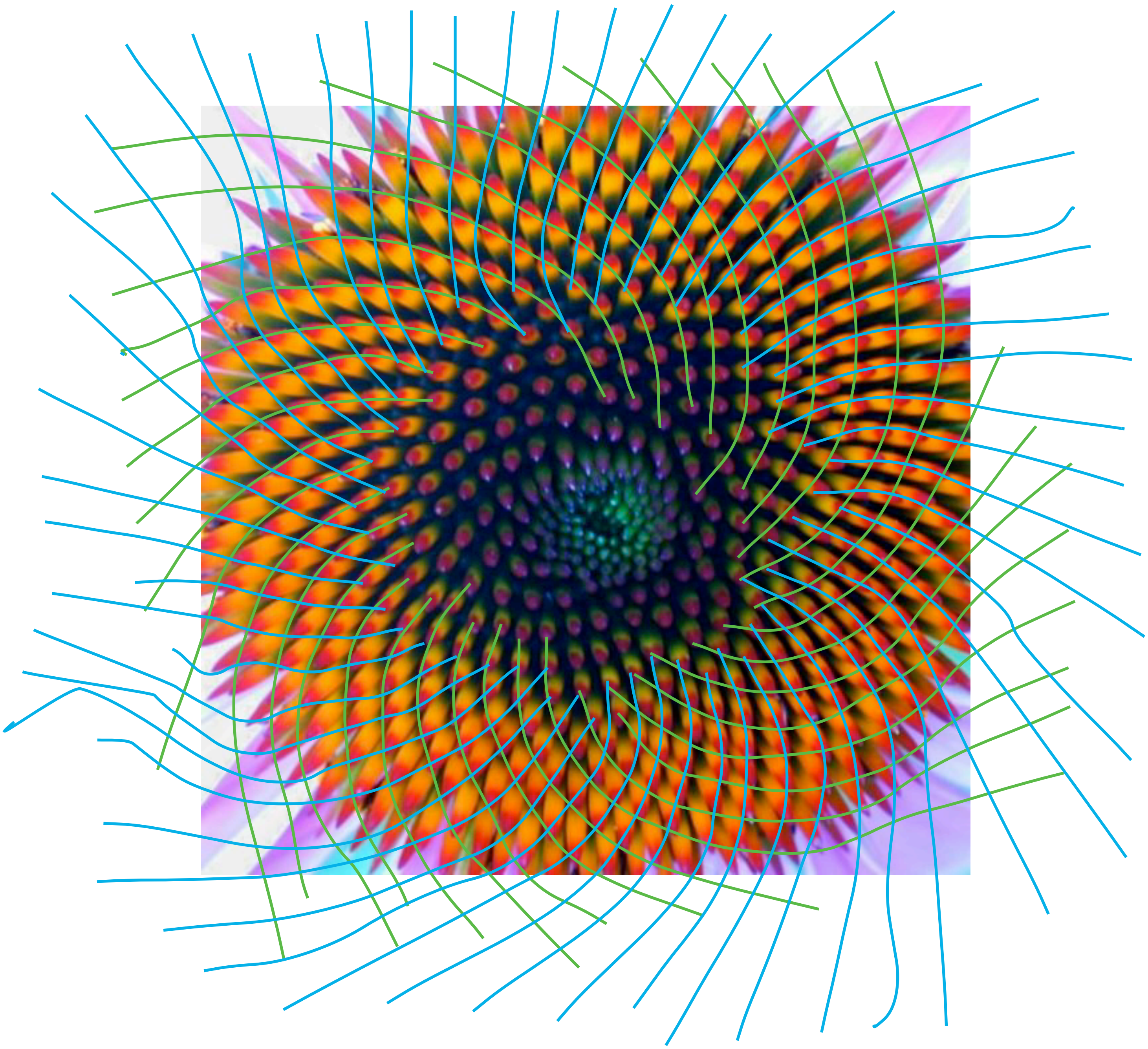

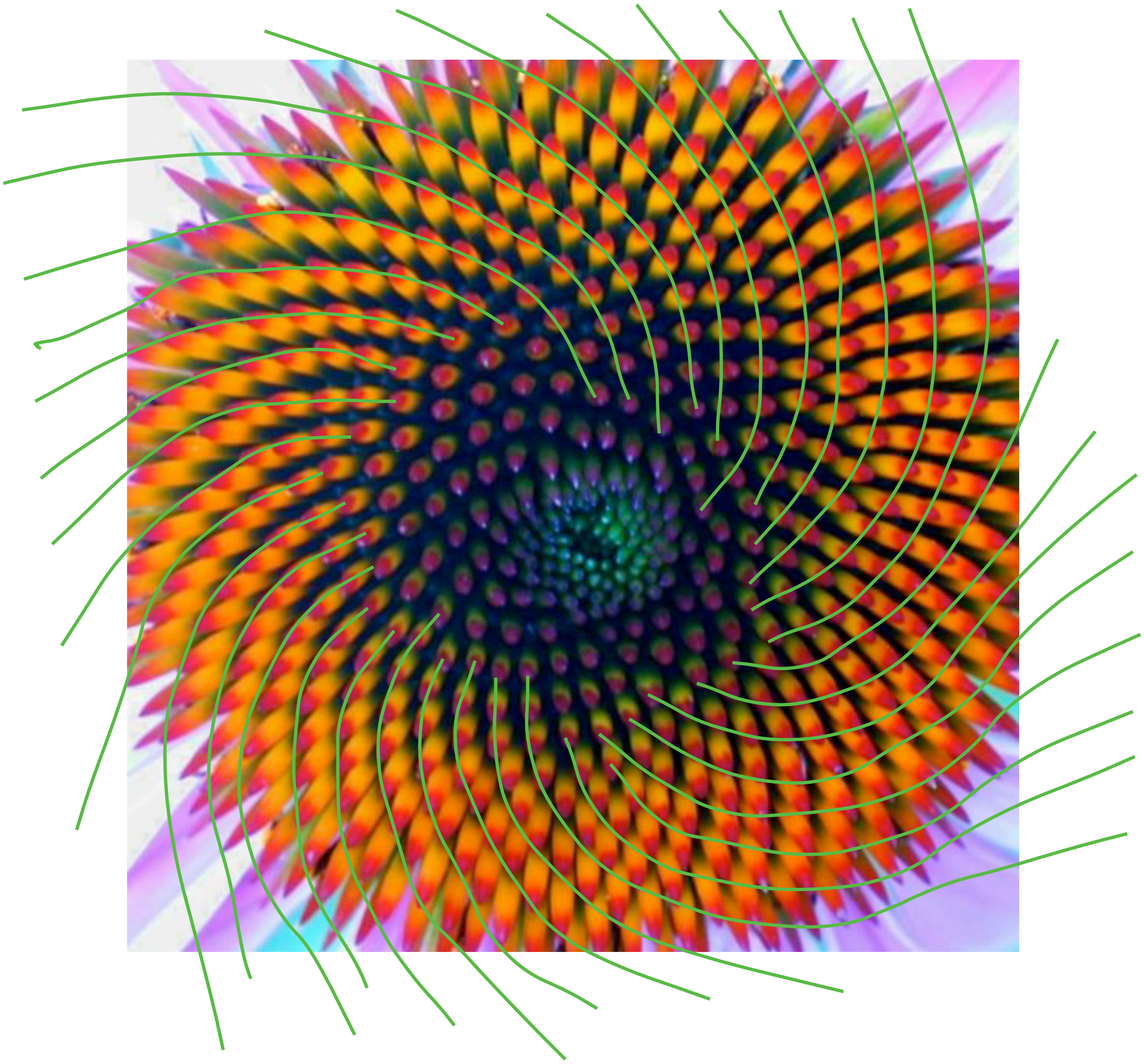

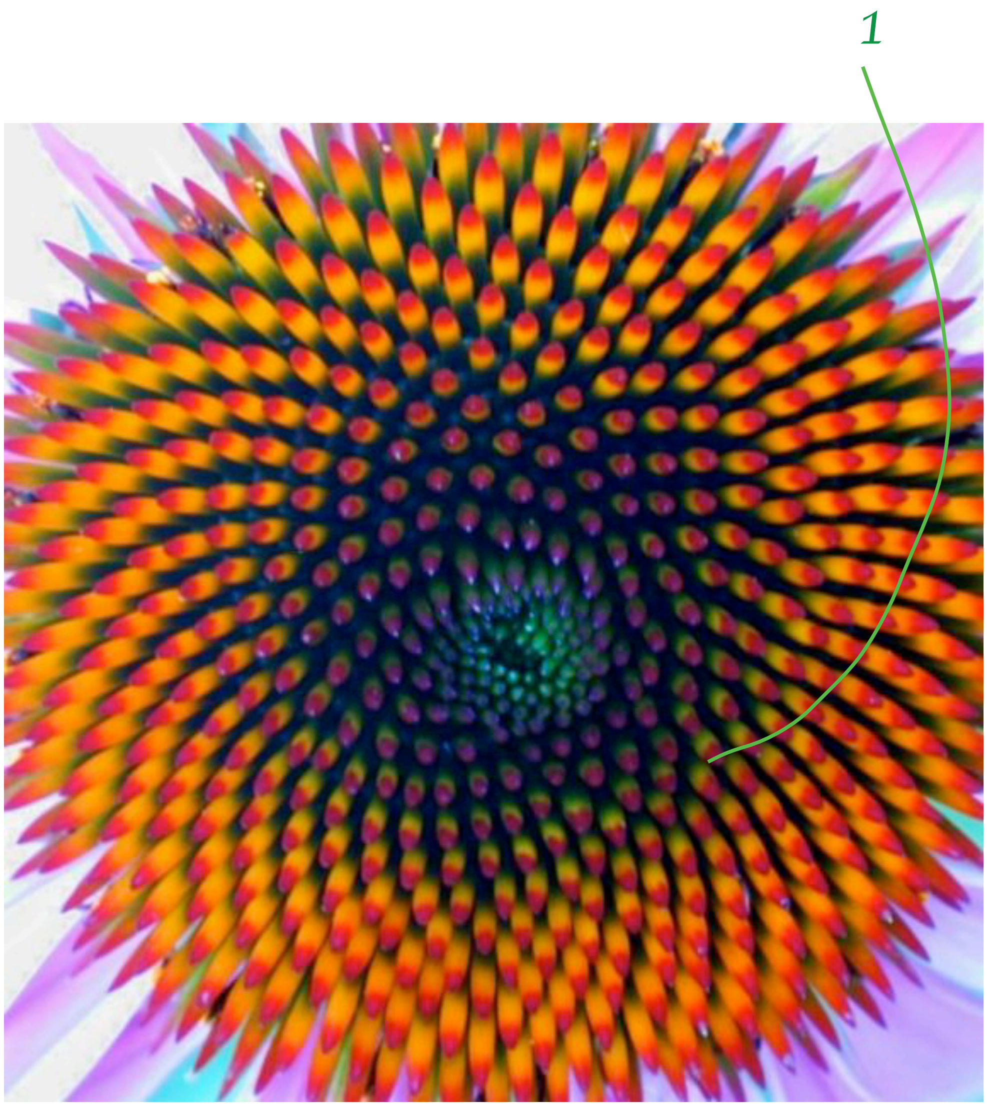
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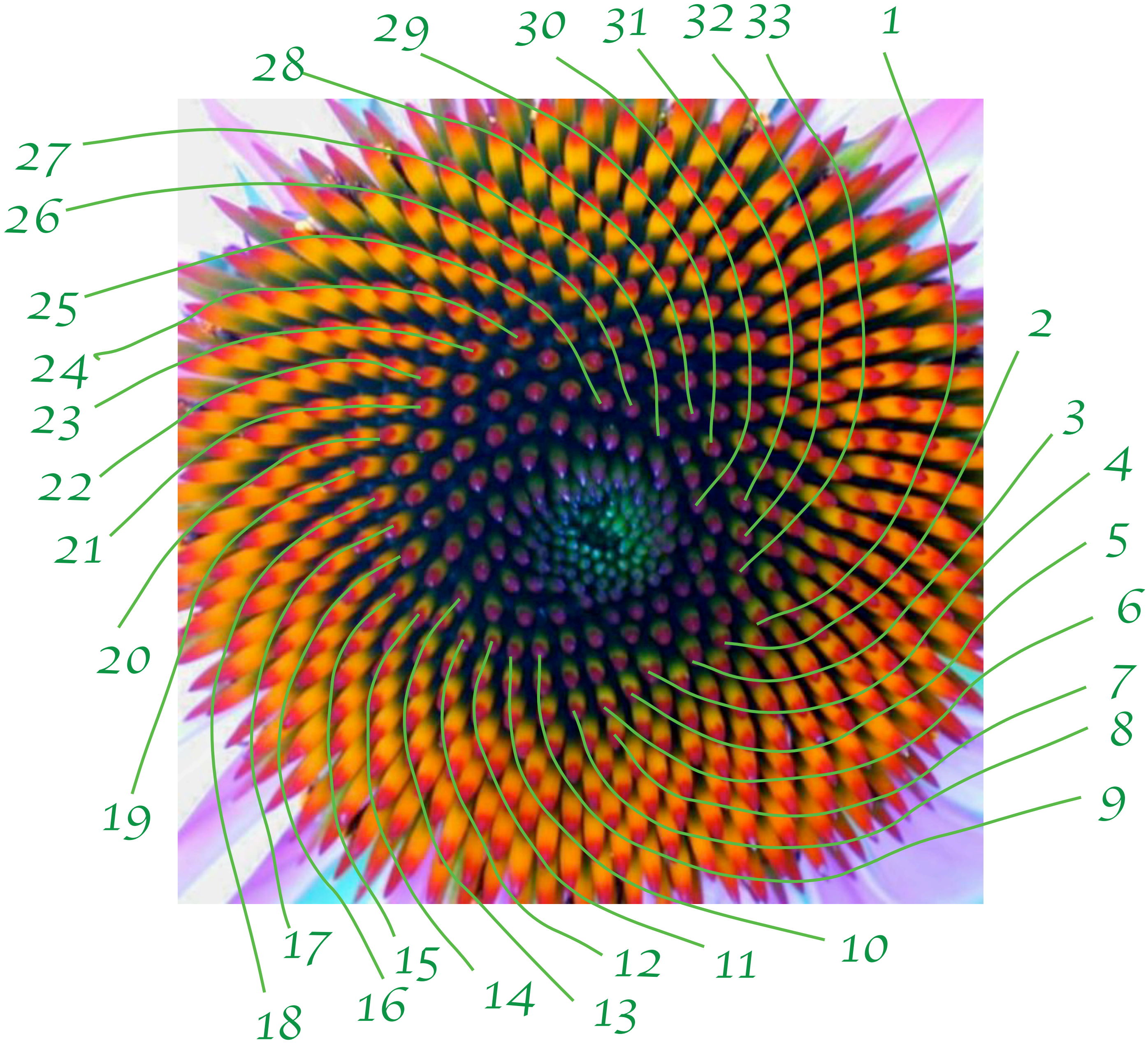
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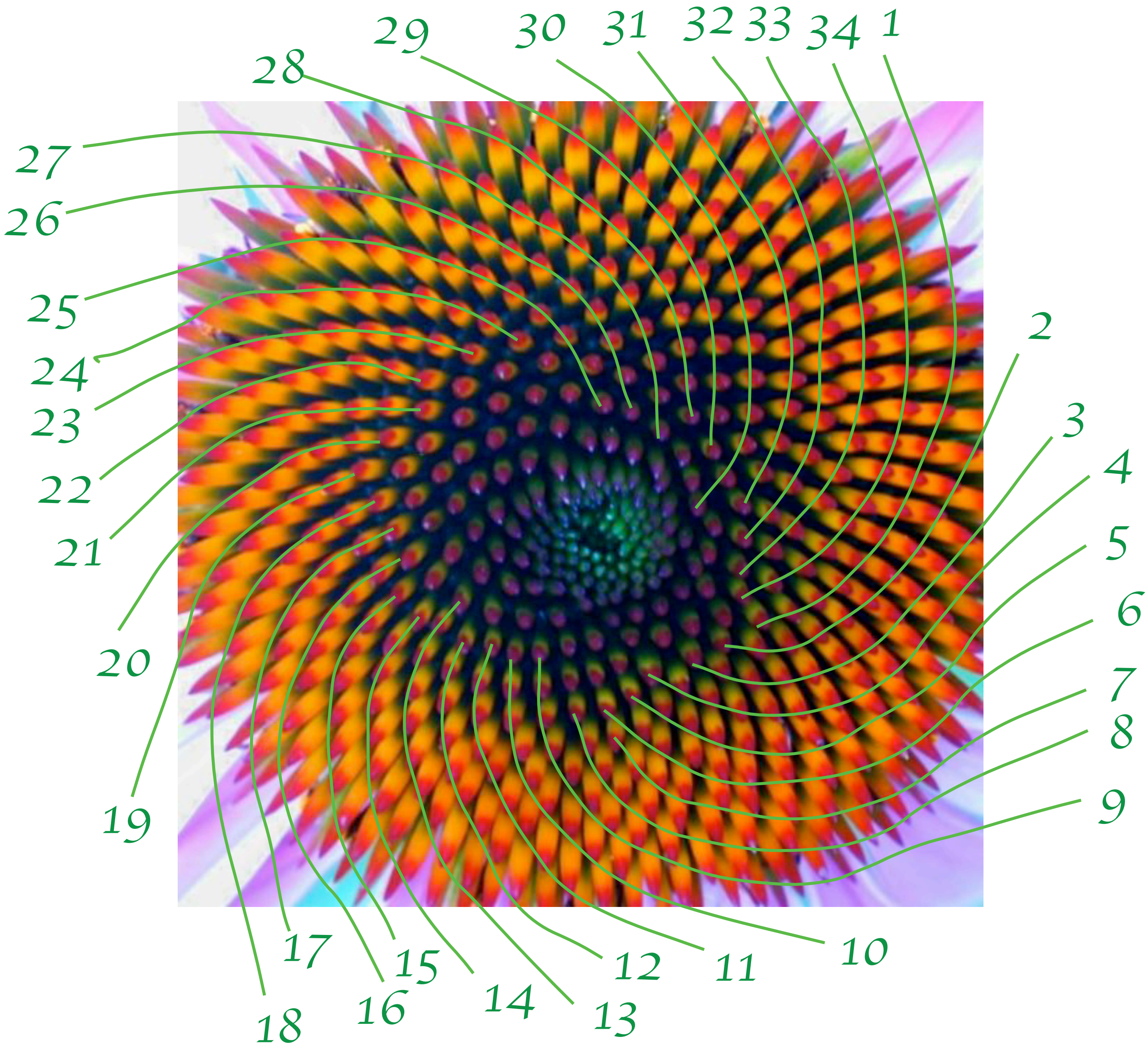

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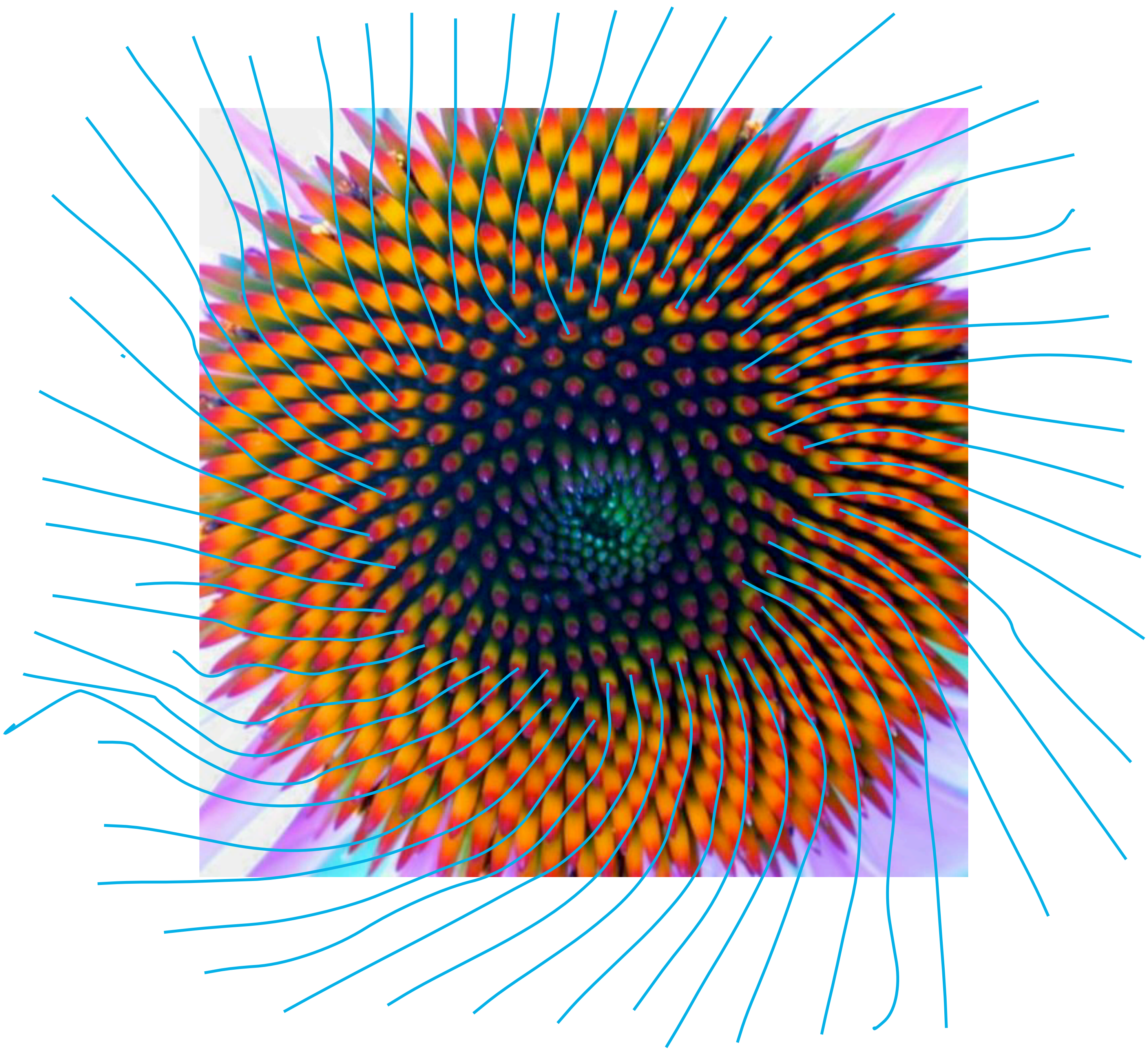

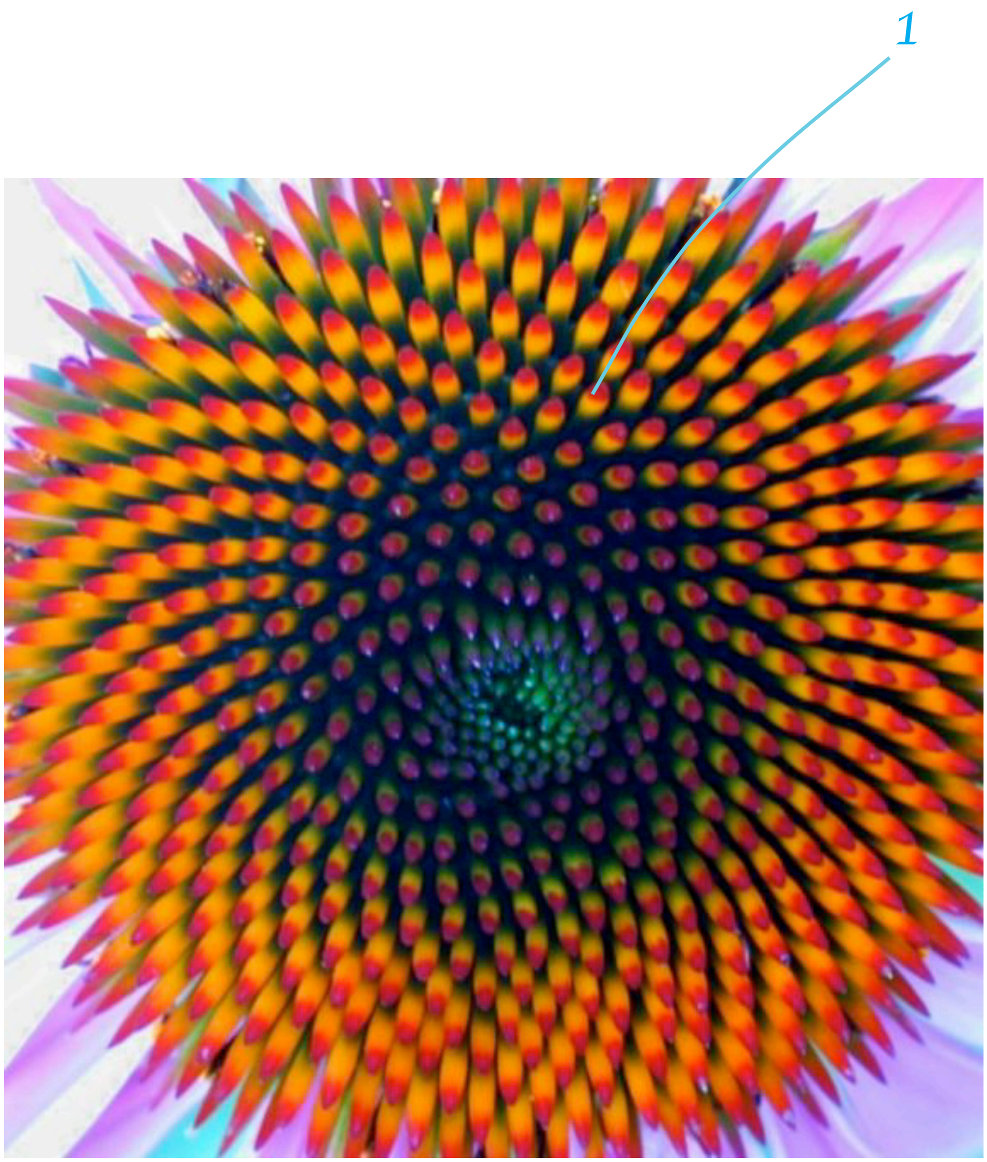
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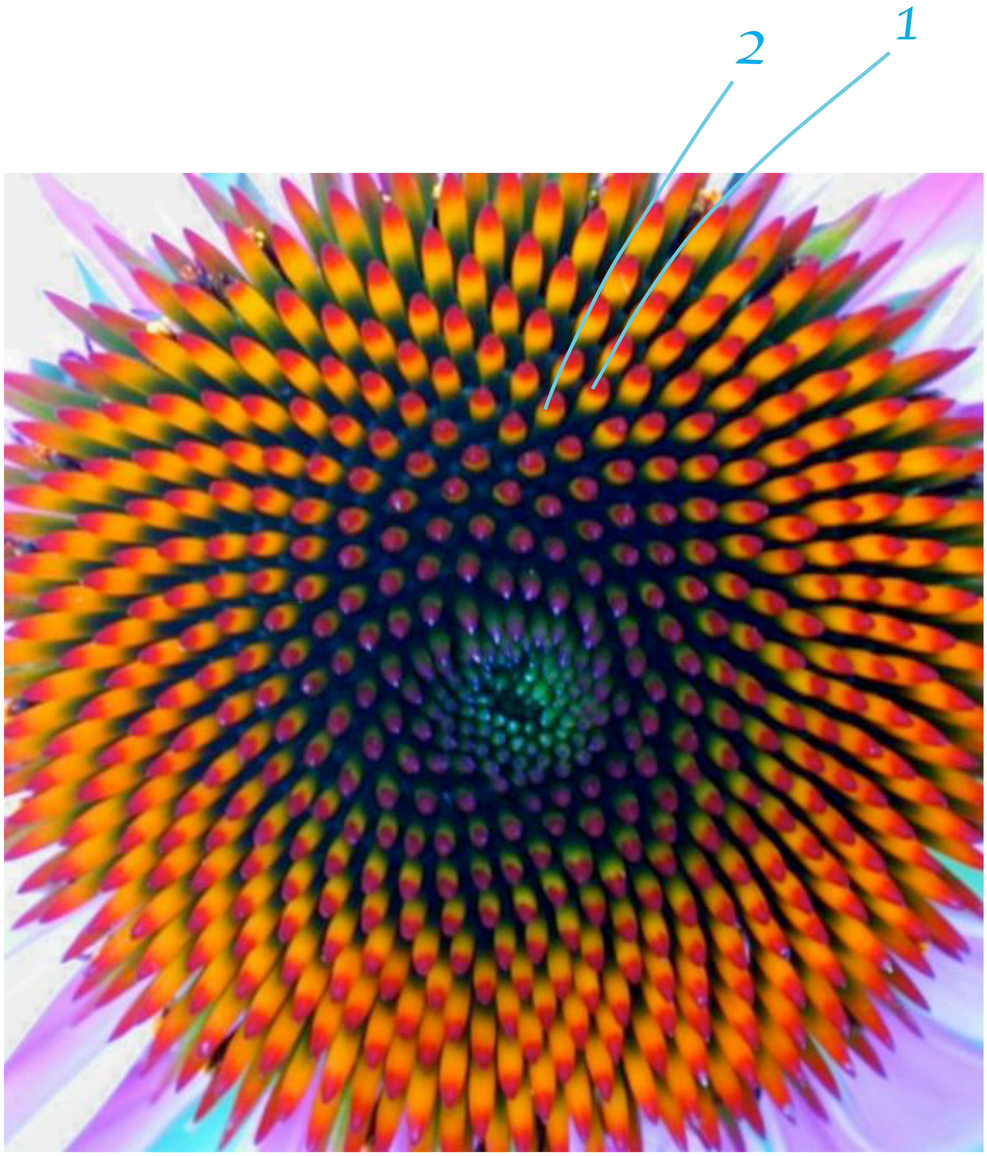
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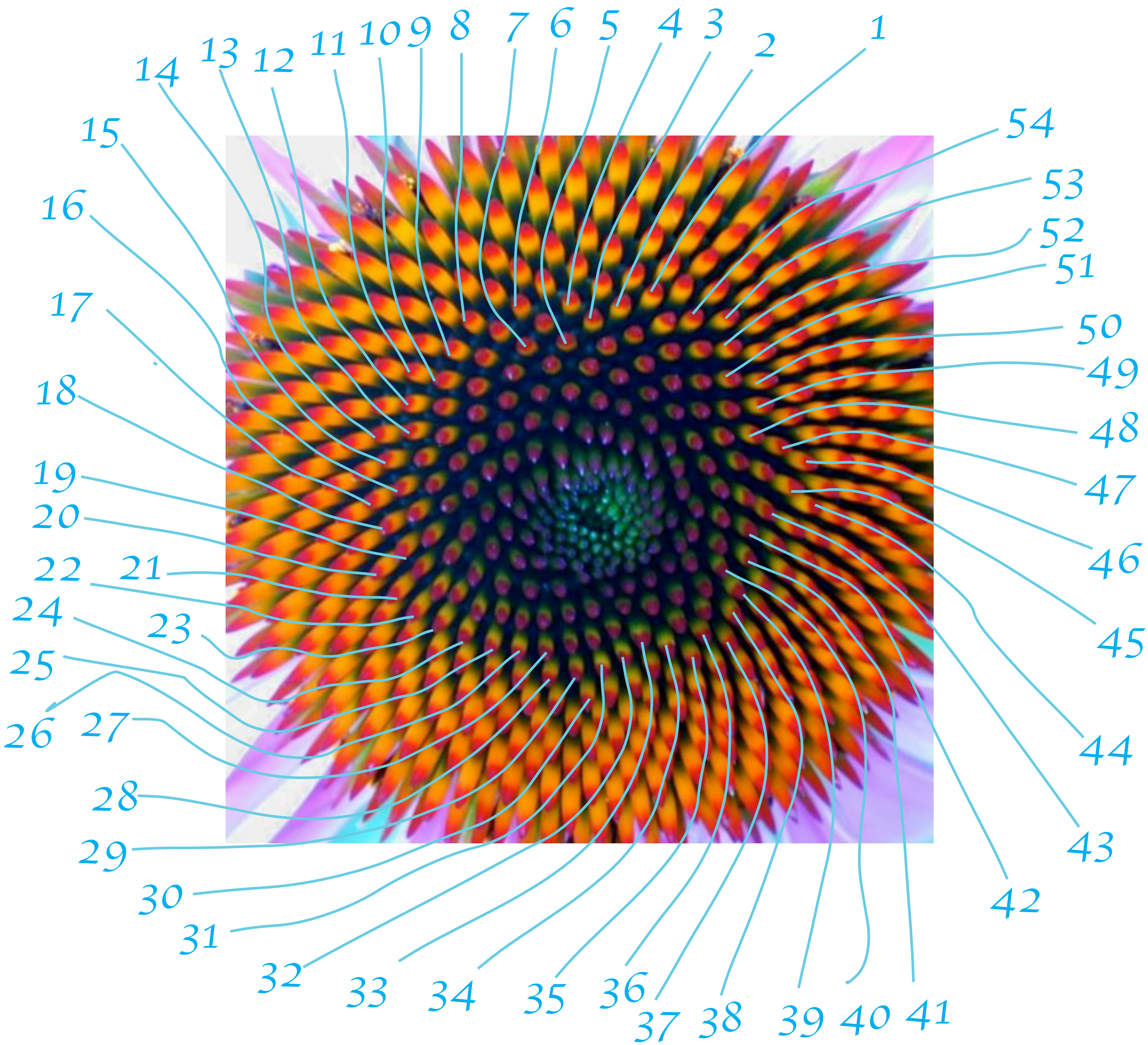
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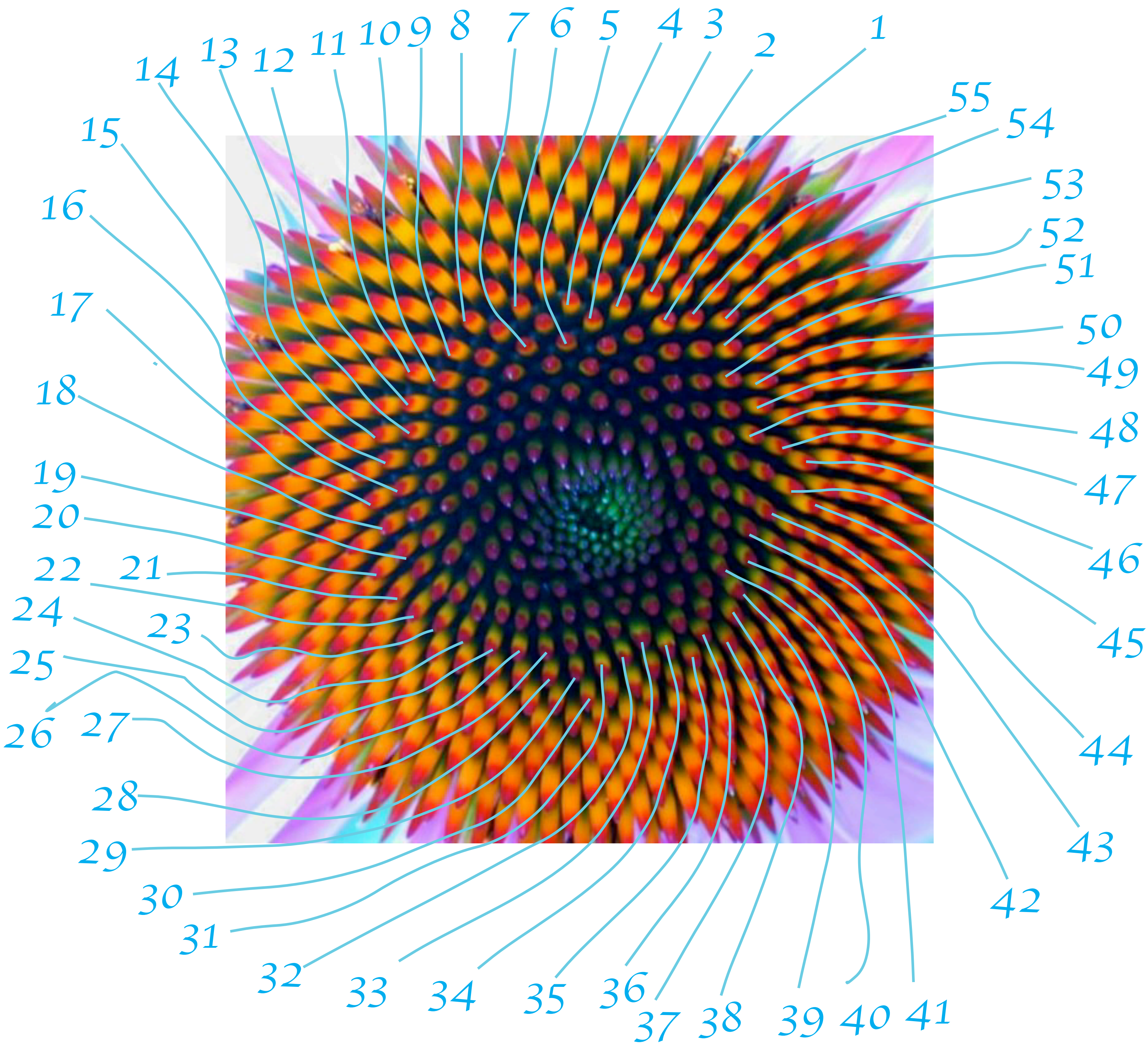
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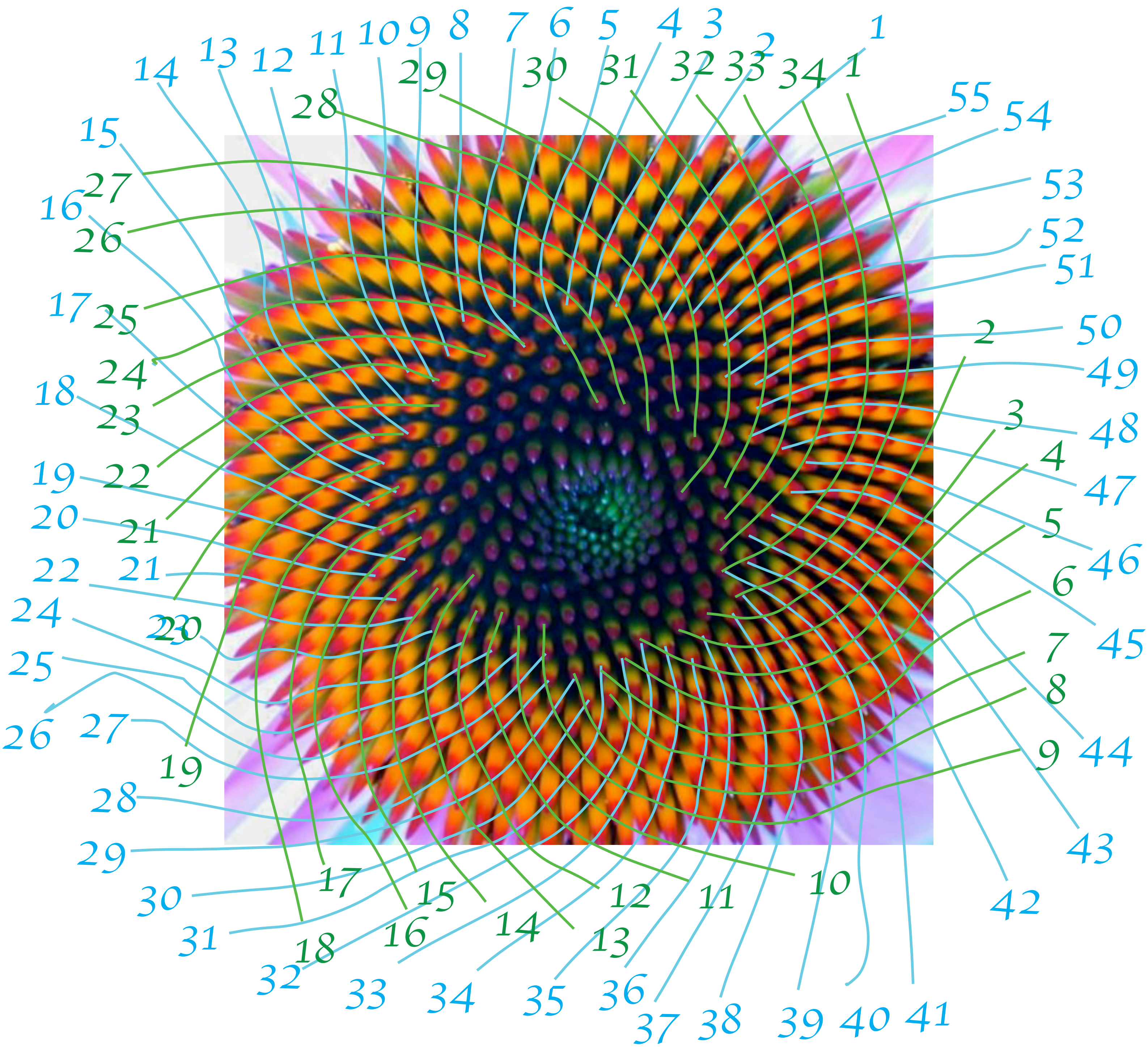

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The Golden Ratio!!!!

1.6180339...

1, 1, 2, 3, 5, 8, 13,
21, 34, 55, 89, 144, ...

1/1 = 1
2/1 = 2
3/2 = 1.5
5/3 = $1.\overline{6}$
8/5 = 1.6
13/8 = 1.625
21/13 = 1.61538
34/21 = 1.619
⋮
144/89 = 1.617

Limit of ratios of successive Fibonacci numbers:

$$R_n = F_n / F_{n-1}$$

$$= \frac{F_{n-1} + F_{n-2}}{F_{n-1}}$$

$$= 1 + \frac{F_{n-2}}{F_{n-1}}$$

$R_n = F_n / F_{n-1}$

$R_{n-1} = F_{n-1} / F_{n-2}$

$1/R_{n-1} = F_{n-2} / F_{n-1}$

$$R_n = 1 + \frac{1}{R_{n-1}}$$

Take limits:

$$\lim_{n\to\infty} R_n = 1 + \frac{1}{\lim_{n\to\infty} R_{n-1}}$$

$\to \tau$

If we call this limit τ, then ...

$$\tau = 1 + \frac{1}{\tau}$$

Solve for τ: $\tau = \Phi$!!

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GOLDEN BLOSSOMS, PI FLOWERS

By Ivars Peterson
Web edition

Enlarge
I. Peterson

In the head of a sunflower, the tiny florets that turn into seeds are typically arranged in two intersecting families of spirals, one winding clockwise and the other winding counterclockwise. Count the number of florets along a spiral and you are likely to find 21, 34, 55, 89, or 144. Indeed, if 34 floret (or seed) rows curve in one direction, there will be either 21 or 55 rows curving in the other direction.

These numbers all belong to a sequence named for the 13th-century Italian mathematician Fibonacci. Each consecutive number is the sum of the two numbers that precede it. Thus, **1** + **1** = **2**, 1 + 2 = **3**, 2 + 3 = **5**, 3 + 5 = **8**, 5 + 8 = **13**, and so on.

The ratios of successive terms of the Fibonacci sequence get closer and closer to a specific irrational number, often called the golden ratio. The golden ratio can be represented as (1 + sqrt[5])/2, or 1.6180339887. . . . Note that the ratio 55/34 is 1.617647. . ., and the next ratio, 89/55, is 1.6181818. . ., and so on.