1.5 Linear functions and proportional models

Changes in input and output

Suppose y is a function of x. Then there is some rule that answers the question: What is the value of y for any given x? Often, however, we start by knowing the value of y for a particular x, and the question we really want to ask is: How does y respond to *changes* in x? We are still dealing with the same function – just looking at it from a different point of view. This point of view is important; we have used it, and will continue to use it, to analyze functions (like $S(t)$, $I(t)$, and $R(t)$) that are defined by rate equations. (This is the essence of *Euler's method.*)

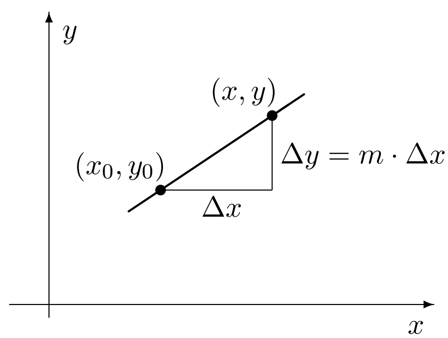
The way Δy depends on Δx can be simple or it can be complex, depending on the function involved. The simplest possibility is that Δy and Δx are **proportional**:

$$\Delta y = m\,\Delta x, \quad \text{for some constant } m.$$

Thus, if Δx is doubled, so is Δy; if Δx is tripled, so is Δy. A function whose input and output are related in this simple way is called a **linear function**, because the graph is a straight line. Let's take a moment to see why this is so.

The graph of a linear function. The graph consists of certain points (x, y) in the x, y-plane. Our job is to see how those points are arranged. Fix one of them, and call it (x_0, y_0). Let (x, y) be any other point on the graph. Draw the line that connects this point to (x_0, y_0), as we have done in the figure at the right. Now set

$$\Delta x = x - x_0, \qquad \Delta y = y - y_0.$$



By definition of a linear function, $\Delta y = m \cdot \Delta x$, as the figure shows, so the slope of this line is $\Delta y/\Delta x = m$. Recall that m is a constant; thus, if we pick a new point (x, y), the slope of the connecting line won't change.

Since (x, y) is an arbitrary point on the graph, what we have shown is that **every point on the graph lies on a line of slope m through the point (x_0, y_0)**. But there is only one such line – and all the points lie on it! That line must be the graph of the linear function.

A linear function is one that satisfies $\Delta y = m \cdot \Delta x$; its graph is a straight line whose slope is m.

Definition of linear function

The above definition tells us that, for a straight line, the ratio $\Delta y/\Delta x$ is *constant*. That is, this ratio does not depend on where the interval of length Δx begins or ends. Note that this is not so for functions that *do not* give straight lines. For more general functions, the ratio $\Delta y/\Delta x$ can vary depending on where the interval in question starts, and how long this interval is.

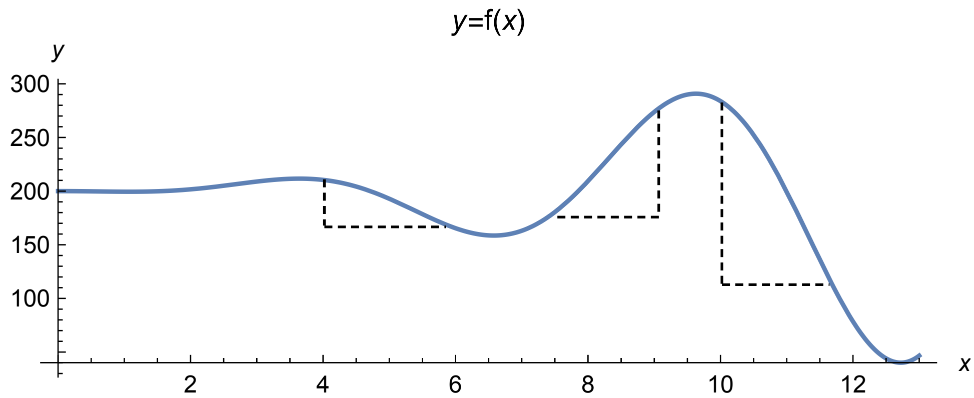


Figure 1.3. For nonlinear functions, $\Delta y/\Delta x$ varies

Rates, slopes, and multipliers. The interpretation of m as a slope is just one possibility; there are two other interpretations that are equally important. To illustrate them we'll use Mark Twain's vivid description of the shortening of the Lower Mississippi River (see page 22). This will also give us the chance to see how a linear function emerges in context.

Twain says "the Lower Mississippi has shortened itself ... an average of a trifle over a mile and a third per year." Suppose we let L denote the length of the river, in miles, and t the time, in years. Then L depends on t, and Twain's statement implies that L is a *linear* function of t – in the sense in which we have just defined a linear function. Here is why. According to our definition, there must be some number m which makes $\Delta L = m \cdot \Delta t$. But notice that Twain's statement has exactly this form if we translate it into mathematical language. Convince yourself that it says

$$\Delta L \text{ miles} = -1\tfrac{1}{3}\ \frac{\text{miles}}{\text{year}} \times \Delta t \text{ years.}$$

Thus we should take m to be $-1\frac{1}{3}$ miles per year.

The role of m here is to convert one quantity (Δt years) into another (ΔL miles) by multiplication. All linear functions work this way. In the defining equation $\Delta y = m \cdot \Delta x$, multiplication by m converts Δx into Δy. Any change in x produces a change in y that is m times as large. For this reason we give m its second interpretation as a **multiplier**.

(It is easier to understand why the usual symbol for *slope* is m – instead of s – when you see that a slope can be interpreted as a multiplier.)

It is important to note that, in our example, m is not simply $-1\frac{1}{3}$; it is $-1\frac{1}{3}$ *miles per year*. In other words, m is the **rate** at which the river is getting shorter. All linear functions work this way, too. We can rewrite the equation $\Delta y = m \cdot \Delta x$ as a ratio

$$m = \frac{\Delta y}{\Delta x} = \text{the rate of change of } y \text{ with respect to } x.$$

For these reasons we give m its third interpretation as a **rate of change**.

For a linear function satisfying $\Delta y = m \cdot \Delta x$,
the coefficient m is
rate of change, slope, and multiplier.

We already use y' to denote the rate of change of y, so we can now write $m = y'$ when y is a linear function of x. In that case we can also write

$$\Delta y = y' \cdot \Delta x.$$

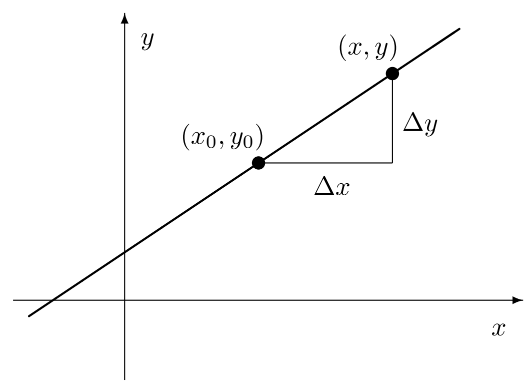
This expression should recall a pattern very familiar to you – see the prediction equation (PEPV), from Section 1.1, and the variants on (PEPV), such as (PEAV), etc. It is the fundamental formula we have been using to calculate (approximate) future values of S, I, and R. We can approach the relation between y and x the same way. That is, if y_0 is an "initial value" of y, when $x = x_0$, then *any* value of y can be calculated from

$$y = y_0 + y' \cdot \Delta x \quad \text{or} \quad y = y_0 + m \cdot \Delta x.$$

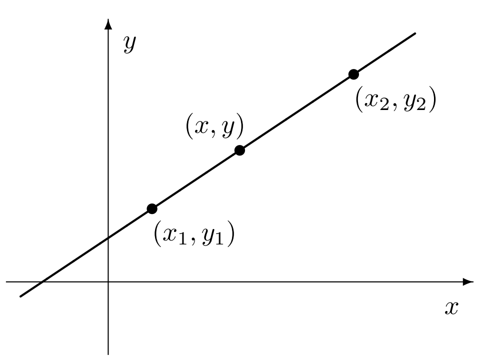
Formulas for linear functions. The expression $\Delta y = m \cdot \Delta x$ declares that y is a linear function of x, but it doesn't quite tell us what y itself *looks like* directly in terms of x. In fact, there are several equivalent ways to write the relation $y = f(x)$ in a formula, depending on what information we are given about the function.

• **The initial-value form**. Here is a very common situation: we know the value of y at an "initial" point – let's say $y_0 = f(x_0)$ – and we know the rate of change – let's say it is m. Then the graph is the straight line of slope m that passes through the point (x_0, y_0). The formula for f is

$$y = y_0 + \Delta y = y_0 + m \cdot \Delta x = y_0 + m(x - x_0) = f(x). \tag{1.5.1}$$



What you should note particularly about this formula is that it expresses y in terms of the initial data x_0, y_0, and m – as well as x. Since that data consists of a point (x_0, y_0) and a slope m, the initial-value formula is also referred to as the **point-slope form** of the equation of a line. It may be more familiar to you with that name.



• **The interpolation form**. This time we are given the value of y at *two* points – let's say $y_1 = f(x_1)$ and $y_2 = f(x_2)$. The graph is the line that passes through (x_1, y_1) and (x_2, y_2), and its slope is therefore

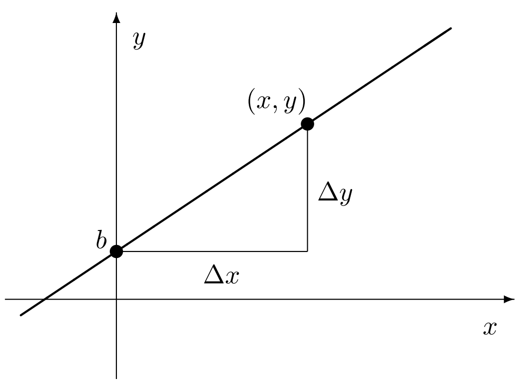
$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Now that we know the slope of the graph we can use the point-slope form (taking (x_1, y_1) as the "point", for example) to get the equation. We have

$$y = y_1 + m(x - x_1) = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) = f(x). \tag{1.5.2}$$

Notice how, once again, y is expressed in terms of the initial data – which consists of the two points (x_1, y_1) and (x_2, y_2).

The process of finding values of a quantity between two given values is called **interpolation**. Since our new expression does precisely that, it is called the interpolation formula. (Of course, it also finds values outside the given interval.) Since the initial data is a pair of points, the interpolation formula is also called the **two-point formula** for the equation of a line.



• **The slope-intercept form**. This is a special case of the initial-value form that occurs when the initial $x_0 = 0$. Then the point (x_0, y_0) lies on the y-axis, and it is frequently written in the alternate form $(0, b)$. The number b is called the ***y*-intercept**. The equation is

$$y = mx + b = f(x). \tag{1.5.3}$$

In the past you may have thought of this as *the* formula for a linear function, but for us it is only one of several. You will find that we will use the other forms more often.

Example 1.5.1. Find an equation for each line with the given properties. Then put your equation into slope-intercept form.

(a) The line through (−1,2) and (−6,4).

(b) The line through (3,1), and such that every two units change in x produces −4 units change in y.

(c) The line through (0,2) with slope 2/3.

Solution. (a) We use the interpolation, or two-point, formula (1.5.2); we find that

$$y = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) = 2 + \frac{4-2}{-6-(-1)}(x-(-1)) = 2 + \frac{2}{-5}(x-(-1)) = 2 - \frac{2}{5}(x+1).$$

To put this into slope-intercept form, we just simplify:

$$y = 2 - \frac{2}{5}(x+1) = 2 - \frac{2}{5}x - \frac{2}{5} = -\frac{2}{5}x + \frac{8}{5}.$$

(So the slope is $m = -2/5$; the y-intercept is $b = 8/5$.) Note that, for this computation, we made a choice of which point to think of as (x_1,y_1), and which to think of as (x_2,y_2). But, had we made the other choice, we would (in general; not just in this example) have obtained the same answer:

$$y = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) = 4 + \frac{2-4}{-1-(-6)}(x-(-6)) = 2 + \frac{-2}{5}(x-(-6)) = 4 - \frac{2}{5}(x+6)$$
$$= 4 - \frac{2}{5}x - \frac{12}{5}x = -\frac{2}{5}x + \frac{8}{5}.$$

(b) This line has slope $m = -4/2 = -2$, and therefore, by the point-slope formula (1.5.1), has equation

$$y = y_0 + m(x - x_0) = 1 + (-2)(x - 3) = 1 - 2(x - 3)$$

or, in slope-intercept form,

$$y = 1 - 2x + 6 = -2x + 7.$$

(c) The equation is

$$y = \frac{2}{3}x + 2.$$

The upshot of the above discussions is that, when the change in one quantity is proportional to the change in another, then the relation between these quantities follows a *linear model*. Here's an example.

Example 1.5.2. Thermal Expansion. Measurements show that the length L of a metal bar increases in proportion to the increase in temperature T. An aluminum bar that is exactly 100 inches long when the temperature is 40°F becomes 100.0052 inches long when the temperature increases to 80°F.

(a) How long is the bar when the temperature is 60°F? 100°F?

(b) What is the multiplier m that connects an increase in length ΔL to an increase in temperature ΔT?

(c) Express ΔL as a linear function of ΔT.

(d) How long will the bar be when $T = 0$°F?

(e) Express L as a linear function of T.

(f) What temperature change would make $L = 100.01$ inches?

(g) What is the multiplier μ that connects an increase in temperature ΔT to an increase in length ΔL? (The symbol μ is the Greek letter *mu*.)

(h) Express T as a linear function of L.

Solution. (a) We are told that the change in length is proportional to the change in temperature. A 40° F increase in temperature – from 40° F to 80° F – produces a length change of $100.0052 - 100 = 0.0052$ in (inches). So half that temperature increase – from 40° F to 60° F – will produce half the increase in length, meaning an increase of $0.0052/2 = 0.0026$ in. So at 60° F, the length of the bar will be $100 + 0.0026 = 100.0026$ in.

(b) In part (a) we observed that, when $\Delta T = 40°$ F, we have $\Delta L = 0.0052$ in. So the equation $\Delta L = m\,\Delta T$ satisfies 0.0052 in $= m \times 40°$ F. Solving for m gives

$$m = \frac{0.0052 \text{ in}}{40° \text{ F}} = 0.00013 \,\frac{\text{in}}{°\text{F}}.$$

(c) $\Delta L = 0.00013\,\Delta T$.

(d) If $\Delta T = -40°$ F, then $\Delta L = 0.00013 \times (-40) = -0.0052$ in, which implies a length of $L = 100 + (-0.0052) = 99.9948$ in, when $T = 0°$ F . (The bar shrinks just as much, for a temperature *drop* of 40° F, as it expands for a temperature *gain* of 40° F.)

(e) The line expressing L in terms of T has slope $m = 0.00013$ and L-intercept $b = 99.9948$, and therefore, by the slope-intercept formula (1.5.3), has equation

$$L = 0.00013T + 99.9948. \tag{1.5.4}$$

(f) We solve $100.01 = 0.00013T + 99.9948$ for T, to get

$$T = \frac{100.01 - 99.9948}{0.00013} = 116.9231° \text{ F}.$$

(g) Solving $\Delta L = 0.00013\,\Delta T$ for ΔT gives $\Delta T = \Delta L/0.00013 = 7692.3077\,\Delta L$. So $\mu = 7692.3077$ degrees Fahrenheit per inch.

(h) We can use the point-slope formula (1.5.1). Or we can solve (1.5.4) for T, to get

$$T = \frac{L - 99.9948}{0.00013} = 7692.3077L - 769190.7692.$$

Proportionality in rate equations

We have been discussing situations where a change in one quantity is proportional to the change in another. A somewhat analogous, though mathematically quite different, situation is where the *rate of change* of a quantity is proportional to that quantity itself.

Consider a human population as an example. If a city of 100,000 persons is increasing at the rate of 1,500 persons per year, we might expect a similar city of 200,000 persons to be increasing at the rate of 3,000 persons per year. That is, we might expect the rate P' of growth of a population to be proportional to the size P of the population P. In symbols,

$$\boxed{P' = kP}$$

A quantity proportional to its rate of change

In the present circumstance, we would have $k > 0$, since the population is growing rather than shrinking.

Note that solving the above equation for k gives

$$k = \frac{P'}{P}.$$

So k is the growth rate divided by the number of individuals; that is, k is the growth rate *per individual*. For this reason, k is often called the *per capita* growth rate. ("Per capita" literally means "per head.")

In particular, the units of k are those of P' divided by those of P. So, for example, if P is measured in persons and time t in years, then the units for k are

$$\frac{\text{persons/year}}{\text{person}}, \quad \text{or simply 1/year (or year}^{-1}\text{)}.$$

In a later chapter, we will see that the solution $P(t)$ to such a rate equation is an *exponential* function of t. In the meantime, we study such rate equations using Euler's method.

Example 1.5.3. In 1985, the per capita growth rate in Poland was 9 persons per year per thousand persons. Assuming that the population of Poland grows in the manner described above:

(a) Let P denote the population of Poland. Write a rate equation for P' in terms of P.

(b) In 1985, the population of Poland was estimated to be 37.5 million persons. What was the net growth rate P' (as distinct from the *per capita* growth rate) in 1985?

(c) Use Euler's method, with $\Delta t = 3$ months, to estimate the population of Poland in 1986.

(d) About how long did it take the population to increase by one person in Poland in 1985?

Solution. (a) We have $P' = kP$. But the given information tells us that, in 1985, $P'/P = 9/1000 = 0.009$, so $k = 0.009$, so

$$P' = 0.009P.$$

Here, P is in persons, P' in persons per year, and k in persons per year per year, or year^{-1}.

(b) In 1985,

$$P' = 0.009 \times 37.5 = 0.3375$$

million persons per year.

(c) In years, we have $\Delta t = 1/4 = 0.25$. We take 1985 as $t = 0$; then Euler's method gives

$$\begin{aligned}
P(1/4) &= P(0) + \Delta P \approx P(0) + P'(0)\Delta t \\
&= P(0) + (kP(0))\Delta t = 37.5 + (0.009 \times 37.5 \times 0.25) = 37.5844, \\
P(1/2) &= P(1/4) + \Delta P \approx P(1/4) + P'(1/4)\Delta t \\
&\approx P(1/4) + (kP(1/4))\Delta t = 37.5844 + (0.009 \times 37.5844 \times 0.25) = 37.6690, \\
P(3/4) &= P(1/2) + \Delta P \approx P(1/2) + P'(1/2)\Delta t \\
&\approx P(1/2) + (kP(1/2))\Delta t = 37.6690 + (0.009 \times 37.6690 \times 0.25) = 37.7538, \\
P(1) &= P(3/4) + \Delta P \approx P(3/4) + P'(3/4)\Delta t \\
&\approx P(3/4) + (kP(3/4))\Delta t = 37.7538 + (0.009 \times 37.7538 \times 0.25) = 37.8387
\end{aligned}$$

million persons.

(d) We have $\Delta P \approx P'\Delta t$. So by part (b) we find that, in 1985, a ΔP of one person corresponds to an elapsed time of

$$\Delta t \approx \frac{\Delta P}{P'} = \frac{1}{0.3375 \times 10^6} = 2.9630 \times 10^{-6}$$

years, or

$$2.9630 \times 10^{-6} \text{ yr} \times 365 \frac{\text{day}}{\text{yr}} \times 24 \frac{\text{hr}}{\text{day}} \times 60 \frac{\text{min}}{\text{hr}} \times 60 \frac{\text{sec}}{\text{min}} = 93.4412 \text{ sec}.$$

Exercises

Part 1: Linear functions and graphs

1. Sketch, using a computer, the graph of each of the following functions. Label each axis, and mark a scale of units on it. For each line that you draw, indicate (i) its slope; (ii) its y-intercept; (iii) its x-intercept (where it crosses the x-axis).

(a) $y = -\frac{1}{2}x + 3$

(b) $y = (2x - 7)/3$

(c) $5x + 3y = 12$

2. Using a computer, graph the function $f(x) = .6x + 2$ on the interval $-4 \leq x \leq 4$.

(a) What is the y-intercept of this graph? What is the x-intercept?

(b) Read from the graph the value of $f(x)$ when $x = -1$ and when $x = 2$. What is the difference between these output values? What is the difference between the x values? According to these differences, what is the slope of the graph? According to the *formula*, what is the slope?

3. Go back to the three functions given in problem 1. For each function, choose an initial value x_0 for x, find the corresponding value y_0 for y, and express the function in the form $y = y_0 + m(x - x_0)$.

4. Find an equation for each line with the given properties. Then put your equation into slope-intercept form.

(a) The line through (0, – 2) with slope 5.

(b) The line through (3,7) and (6, – 2).

(c) The line through (3,1), and such that every decrease of one unit in y produces a three unit increase in x.

5. You should be able to answer all parts of this problem without ever finding the equations of the functions involved.

(a) Suppose $y = f(x)$ is a linear function with multiplier $m = 3$. If $f(2) = -5$, what is $f(2.1)$? $f(2.0013)$? $f(1.87)$? $f(922)$?

(b) Suppose $y = G(x)$ is a linear function with multiplier $m = -2$. If $G(-1) = 6$, for what value of x is $G(x) = 8$? $G(x) = 0$? $G(x) = 5$? $G(x) = 491$?

(c) Suppose $y = h(x)$ is a linear function with $h(2) = 7$ and $h(6) = 9$. What is $h(2.046)$? $h(2+a)$?

Part 2: Linear models

6. In Massachusetts there is a sales tax of 5%. The tax T, in dollars, is proportional to the price P of an object, also in dollars. The constant of proportionality is $k = 5\% = .05$. Write a formula that expresses the sales tax as a linear function of the price, and use your formula to compute the tax on a television set that costs \$289.00 and a toaster that costs \$37.50.

7. Suppose $W = 213 - 17\,Z$. How does W change when Z changes from 3 to 7; from 3 to 3.4; from 3 to 3.02? Let ΔZ denote a change in Z and ΔW the change thereby produced in W. Is $\Delta W = m\,\Delta Z$ for some constant m? If so, what is m?

8. (a) In the following table, q is a linear function of p. Fill in the blanks in the table.

p	-3	0		7	13		π
q	7		4	1		0	

(b) Find a formula to express Δq as a function of Δp, and another to express q as a function of p.

9. **Thermometers**. There are two scales in common use to measure the temperature, called **Fahrenheit degrees** and the **Celsius degrees**. Let F and C, respectively, be the temperature on each of these scales. Each of these quantities is a linear function of the other; the relation between them in determined by the following table:

physical measurement	C	F
freezing point of water	0	32
boiling point of water	100	212

(a) Which represents a larger change in temperature, a Celsius degree or a Fahrenheit degree?

(b) How many Fahrenheit degrees does it take to make the temperature go up one Celsius degree? How many Celsius degrees does it take to make it go up one Fahrenheit degree?

(c) What is the multiplier m in the equation $\Delta F = m \cdot \Delta C$? What is the multiplier μ in the equation $\Delta C = \mu \cdot \Delta F$? What is the relation between μ and m?

(d) Express F as a linear function of C. (We have already done this in an earlier section. Try to do it again "from scratch," using only the previous parts of this exercise.)

(e) Express C as a linear function of F.

(f) Is there any temperature that has the same reading on the two temperature scales? What is it? Does the temperature of the air ever reach this value? Where?

10. **The Greenhouse Effect**. The concentration of carbon dioxide (CO_2) in the atmosphere is increasing. The concentration is measured in parts per million (PPM). Records kept at the South Pole show an increase of .8 PPM per year during the 1960s.

(a) At that rate, how many years does it take for the concentration to increase by 5 PPM; by 15 PPM?

(b) At the beginning of 1960 the concentration was about 316 PPM. What would it be at the beginning of 1970; at the beginning of 1980?

(c) Draw a graph that shows CO_2 concentration as a function of the time since 1960. Put scales on the axes and label everything clearly.

(d) The *actual* CO_2 concentration at the South Pole was 324 PPM at the beginning of 1970 and 338 PPM at the beginning of 1980. Plot these values on your graph, and compare them to your calculated values.

(e) Using the actual concentrations in 1970 and 1980, calculate a new rate of increase in concentration. Using that rate, estimate what the increase in CO_2 concentration was between 1970 and 1990. Estimate the CO_2 concentration at the beginning of 1990.

(f) Using the rate of .8 PPM per year that held during the 1960s, determine how many years before 1960 there would have been *no* carbon dioxide at all in the atmosphere.

11. **Falling Bodies**. In the simplest model of the motion of a falling body, the velocity increases in proportion to the increase in the time that the body has been falling. If the velocity is given in feet per second, measurements show the constant of proportionality is approximately 32.

(a) A ball is falling at a velocity of 40 feet/sec after 1 second. How fast is it falling after 3 seconds?

(b) Express the change in the ball's velocity Δv as a linear function of the change in time Δt.

(c) Express v as a linear function of t.

The model can be expanded to keep track of the *distance* that the body has fallen. If the distance d is measured in feet, the units of d' are feet per second; in fact, $d' = v$. So the model describing the motion of the body is given by the rate equations

$$\begin{aligned} d' &= v \quad \text{feet per second;} \\ v' &= 32 \quad \text{feet per second per second.} \end{aligned}$$

(d) At what rate is the distance increasing after 1 second? After 2 seconds? After 3 seconds?

(e) Is d a linear function of t? Explain your answer.

Part 3: Proportionality in rate equations

For these exercises, you should refer to the subsection "Proportionality in rate equations" of Section 1.5 above.

12. **Afghanistan**. In 1985 the per capita growth rate in Afghanistan was 21.6 persons per year per thousand.

(a) Let A denote the population of Afghanistan. Write the equation that governs the growth rate A' of A.

(b) In 1985 the population of Afghanistan was estimated to be 15 million persons. What was the net growth rate A' in 1985?

(c) Comparing part (b) of this exercise with part (b) of Example (1.5.3), comment on the following assertion: When comparing two countries, the one with the larger per capita growth rate will have the larger net growth rate.

(d) Use Euler's method, with $\Delta t = 4$ months, to estimate the population of Afghanistan in 1986.

(e) About how long did it take the population to increase by one person in Afghanistan in 1985? How does this compare with part (d) of Example (1.5.3)?

13. **Bacterial Growth**. A colony of bacteria on a culture medium grows at a rate proportional to the present size of the colony. When the colony weighed 32 grams it was growing at the rate of 0.79 grams per hour.

(a) Write an equation that links the growth rate B' to the size B of the population. Hint: the per capita growth rate k is *not* equal to 0.79. Rather, k can be found by plugging the given information into your rate equation from part (a).

(b) What are the units for your per capita growth rate in part (a) above?

(c) Use Euler's method with $\Delta t = 1/2$ hour to estimate B after one hour.

14. **Radioactivity**. In radioactive decay, radium slowly changes into lead. If one sample of radium is twice the size of a second lump, then the larger sample will produce twice as much lead as the second in any given time. In other words, the rate of decay is proportional to the amount of radium present. *Decay* means *decrease* in the amount present, so if we denote this amount by R, then we have

$$R' = -kR$$

(note the minus sign), where k is a positive parameter, called the (*per unit*) *decay rate*.

(a) Measurements show that 1 gram of radium decays into lead at the rate of $1/2337$ grams per year. (That is, $R' = -1/2337$ grams per year when $R = 1$ gram.) Using this information, evaluate k. That is, supply a numerical value for k, and also state what the appropriate units for k are.

(b) Using a step size of 10 years, use Euler's method to estimate how much radium remains in a 0.072 gram sample after 40 years.

15. **Cooling**. Suppose a cup of hot coffee is brought into a room at 70°F. It will cool off, and it will cool off *faster* when the temperature difference between the coffee and the room is greater. The

simplest assumption we can make is that the rate of cooling is proportional to this temperature difference (this is called Newton's law of cooling). Let C denote the temperature of the coffee, in °F, and C' the rate at which it is cooling, in °F per minute. The new element here is that C' is proportional, not to C, but to the *difference* between C and the room temperature of 70°F.

(a) Write an equation that relates C' and C. It will contain a proportionality constant k. How did you indicate that the coffee is *cooling* and not *heating up*?

(b) When the coffee is at 180°F it is cooling at the rate of 9°F per minute. What is k?

(c) At what rate is the coffee cooling when its temperature is 120°F?

(d) Estimate how long it takes the temperature to fall from 180°F to 120°F. Then make a better estimate, and explain why it is better.