

Goal: To explore some more ideas about modeling with rate equations and *SIR*.

1. A town of population 100,000 is hit with a measles epidemic, which evolves according to the usual *SIR* equations

$$\begin{aligned}S' &= -a S I, \\I' &= a S I - b I, \\R' &= b I.\end{aligned}$$

This unique strain of the measles is known to last for twelve days.

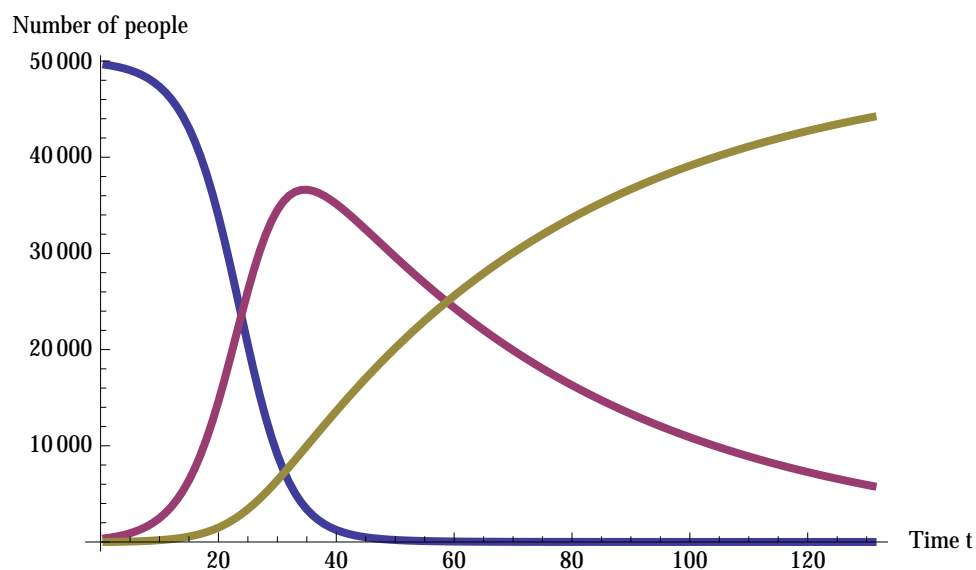
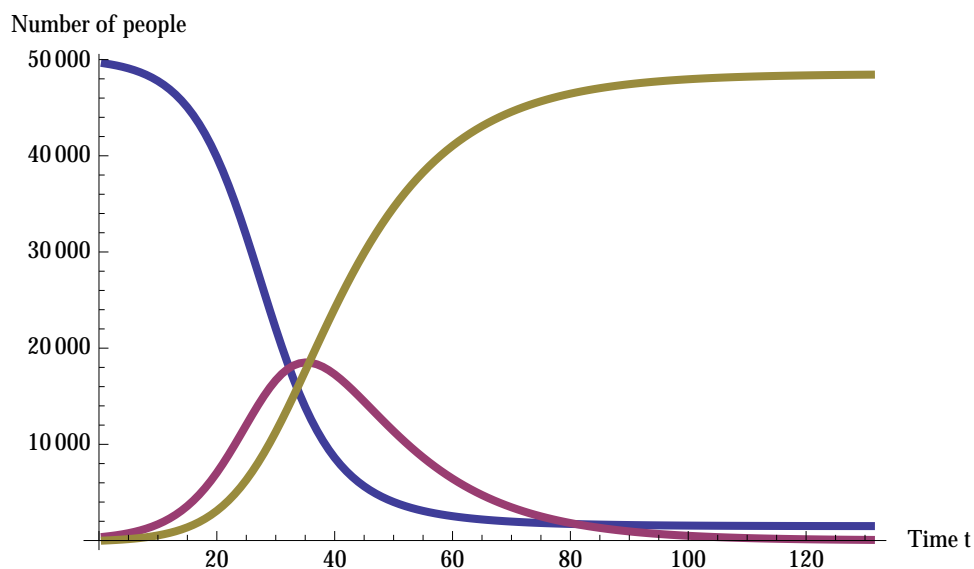
- (a) What is the recovery coefficient b , and what are the units for b ? Please explain. Express b to six decimal places.

On day 15, 14,893 people are susceptible (that is, $S(15) = 14,893$) and 69,613 people are infected (so $I(15) = 69,613$). *One tenth* of a day later, $S = 13,856$.

- (b) What, at least approximately, is $S'(15)$ (the rate of change of S at $t = 15$)? What are the units of $S'(15)$? Hint: this rate of change is, at least roughly, the *net change* in S (final value minus initial value) from day 15 to day 15.1, divided by the elapsed time over that period.

- (c) What is the transmission coefficient a ? What are the units for a ? **HINT:** into above rate equation for S' , plug in S' , S , and I at $t = 15$, and solve for a . Express a to six decimal places.

2. Pictured below are two graphs depicting evolution of diseases that progress according to the usual SIR model. For both graphs, the initial values $S(0)$, $I(0)$ and $R(0)$, and the transmission coefficient a , are the same. But the two graphs correspond to different recovery coefficients b .



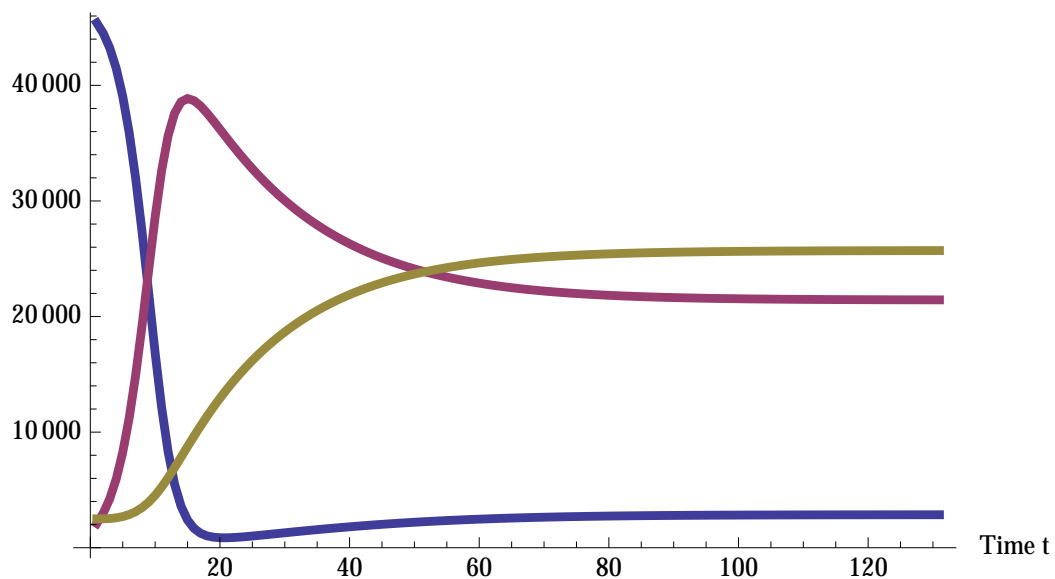
- (a) On each of the graphs, label which curve is S , which is I , and which is R .
- (b) Which of the above two graphs corresponds to the *larger* value of b ? Please explain.

3. Consider an epidemic that progresses according to the usual *SIR* model, *except* that, now, recovered people become susceptible again (and can infect again) after m days.

- (a) *Modify* the usual *SIR* equations to reflect this new feature (wherein recovered can become susceptible again). HINTS: (a) Your new equations will look a *lot* like the old ones, but with some *new terms* added on. These terms should account for the facts that, now, on average, $1/m$ of the recovered population gets *added to* susceptible population, and *subtracted from* the recovered population, on any given day. (b) Your new equations should involve unspecified parameters a , b , and c , where a and b are as above, and $c = 1/m$.

- (b) In the two graphs on the next page, the transmission and recovery coefficients a and b are the same, but the number of days m that it takes to become susceptible again differs from one graph to the next. For which of the two graphs – the one on the top or the one on the bottom — does it take *longer* to become susceptible again? Please explain.

Number of people



Number of people

