## Homework #9: Due 1 PM Thursday, April 16

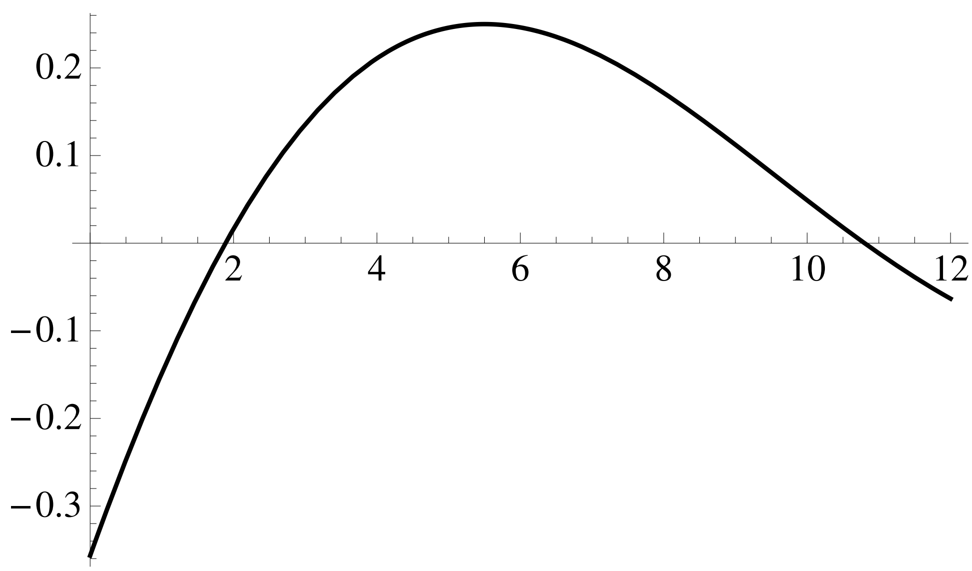
1. If $\sum b_n(x-2)^n$ converges at $x=0$ but diverges at $x=7$, what is the largest possible interval of convergence of this series? What's the smallest possible?

2. (a) Write down the second degree Taylor polynomial $P_2(x)$ approximating
$$f(x) = \ln\big(1 + x(1-x)\big)$$
near $x = 0$.

   (b) Use your result from part (a) to approximate $\ln(1.09)$. Hint: $x = 0.1$.

   (c) What does Taylor's inequality say about the error in the approximation you found in part (b)? You should find it useful to note that
$$f'''(x) = \frac{2(2x-1)\left(x^2-x+4\right)}{\left(x^2-x-1\right)^3},$$
and that $f'''(x)$ is a *decreasing* function on the interval $(0, 0.1)$.

3. Consider the function $y = f(x)$ sketched below.




Suppose $f(x)$ has Taylor series
$$f(x) = a_0 + a_1(x-4) + a_2(x-4)^2 + a_3(x-4)^3 + ...$$
about $x = 4$.

   (a) Is $a_0$ positive or negative? Please explain.

   (b) Is $a_1$ positive or negative? Please explain.

   (c) Is $a_2$ positive or negative? Please explain.

4. How many terms of the Taylor series for $\ln(1+x)$ centered at $x = 0$ do you need to estimate the value of $\ln(1.4)$ to three decimal places?

5. Find the integral and express the answer as an infinite series.
$$\int \frac{e^x - 1}{x}\, dx$$

6. Using series, evaluate the limit
$$\lim_{x \to 0} \frac{\sin x - x}{x^3}.$$

7. Use Taylor's inequality for $P_n(x)$ to find a reasonable bound for the error in approximating the quantity $e^{0.60}$ with a third degree Taylor polynomial for $e^x$ centered at $x = 0$.

8. Consider the error in using the approximation $\sin\theta \approx \theta - \theta^3/3!$ on the interval $[-1, 1]$. Where is the approximation an overestimate? Where is it an underestimate?

9. Find the Taylor series around $x = 0$ for
$$\cosh x = \frac{e^x + e^{-x}}{2}.$$
(Your answer should involve only even powers of $x$.)

10. Suppose the series
$$\sum_{n=5}^{\infty} C_n (x-3)^n$$
converges when $x = 0$ but diverges when $x = 9$. For each of the following values of $x$, determine whether the series converges or diverges there, or if there's not enough information to say. Explain each of your answers briefly.

    (a) $x = 1$

    (b) $x = -5$

    (c) $x = -2$