Stepsize: What's up with that?

Question: how does changing the "stepsize" Δt change (a) computations and (b) results?

Example. Given:

- The usual SIR equations

$$\boxed{\begin{aligned} S' &= -aSI \\ I' &= aSI - bI \\ R' &= bI, \end{aligned}} \quad \text{(SIR)}$$

- Initial conditions
  $S(0) = 500, \ I(0) = 10, \ R(0) = 0,$

- Parameters
  $a = 0.001, \ b = 0.2,$

find $S(4)$, $I(4)$, $R(4)$ using:
(A) stepsize $\Delta t = 4$; (B) stepsize $\Delta t = 2$.

Solution.

(A)
$$\begin{aligned} S(4) &= S(0) + \Delta S \\ &= S(0) + S'(0)\cdot\Delta t \\ &= S(0) + (-a\cdot S(0)\cdot I(0))\cdot\Delta t \\ &= 500 + (-0.001\cdot 500\cdot 10)\cdot 4 \\ &= 500 - 20 = 480. \end{aligned}$$

Also
$$\begin{aligned} R(4) &= R(0) + \Delta R \\ &= R(0) + R'(0)\cdot\Delta t \\ &= R(0) + (b\cdot I(0))\cdot\Delta t \\ &= 0 + (0.2\cdot 10)\cdot 4 = 8. \end{aligned}$$

Then
$$\begin{aligned} I(4) &= S(0) + I(0) + R(0) - S(4) - R(4) \\ &= 500 + 10 + 0 - 480 - 8 = 22. \end{aligned}$$

In sum, $\boxed{S(4) = 480,\ I(4) = 22,\ R(4) = 8}$ people.

(B) We did this last time; we got

$$\boxed{S(4) = 474.32 \quad I(4) = 25.28, \quad R(4) = 10.4} \text{ people.}$$

Notes.

(a) All results are <u>approximate.</u> Why? Because, in equations like

$$S(4) = S(2) + \Delta S = S(2) + S'(2)\Delta t,$$

the second "=" should really be "≈." This is because S' itself typically changes with t, so ΔS is only <u>roughly</u> equal to $S'(t)\Delta t$ for a specific time t.

(b) <u>Smaller</u> Δt means more <u>frequent</u> recalibration of $S'(t)$, which typically means BETTER APPROXIMATIONS.

For example: we could predict $S(4)$, $I(4)$, $R(4)$ using $\Delta t = 0.01$ (and some technology).

After 400 iterations (corresponding to 401 total values of t: $t = 0, 0.01, 0.02, \ldots, 3.99, 4.00$), we get

$$\boxed{S(4) = 463.57, \quad I(4) = 31.30, \quad R(4) = 15.13} \text{ people.}$$

(a <u>better</u> approximation)

Summary: SIR by "Euler's method."