Here we study relationships among numbers of vertices, edges, and faces in a polyhedron.

We begin with some definitions. First: a *simple closed surface* is an object, in three dimensions, that's hollow, has no holes, and does not cross itself anywhere. (That is: you can get from any point inside the surface to any other point inside the surface without crossing any part of the surface itself.) A *polyhedron* is a simple closed surface made up of polygonal regions joined together. (*Polyhedra* is the plural of polyhedron.)

The polygonal regions making up a polyhedron are called *faces* of the polyhedron. The terms *vertices* and *edges*, when applied to a polyhedron, refer simply to the vertices and edges of the polygonal regions making up that polyhedron.

1. First, you need to *build* the polyhedron from the "net" you downloaded. When you're done, COUNT the vertices, edges, and faces of your polyhedron; then enter the name of your polyhedron, as well as the numbers you counted, in the indicated spaces of the first row below.

With your group, complete as many of the rows below as you can. Do this also for any polyhedral objects you have on hand: boxes, dice, etc.

Name of Polyhedron	Number of vertices (V)	Number of edges (E)	Number of faces (F)

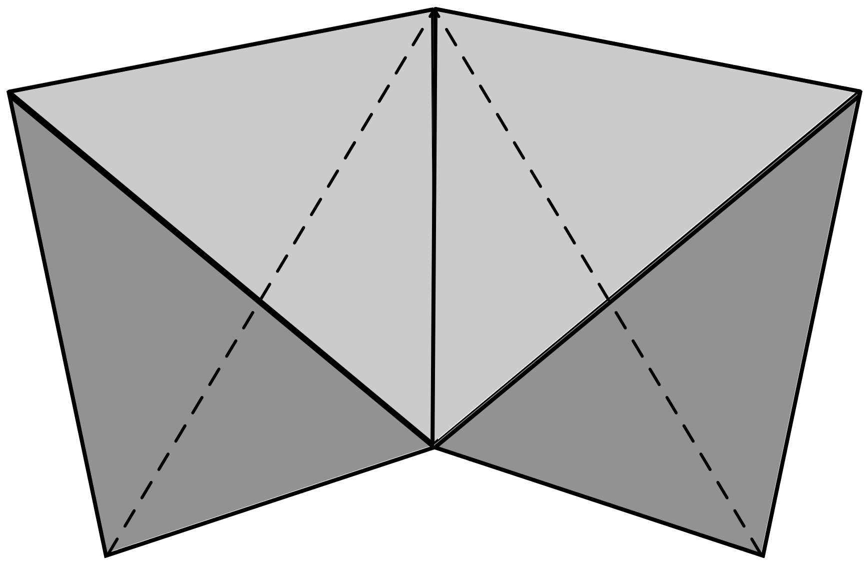
2. Using the information from the table you just completed, answer the following.

CONJECTURE ("Euler's Formula"): If P is any polyhedron, and V, E, and F represent the numbers of vertices, edges, and faces of P, respectively, then

$$\underline{\hspace{3cm}} = 2.$$

(The blank should be filled in with some simple expression involving V, E, and F, which you should be able to guess at by examining your table above.)

3. Use the sketch below of a "dual tetrahedron" (formed by gluing two tetrahedra together along an edge of each) to answer the given questions.



How many vertices does the dual tetrahedron have? _________ How many edges? __________
How many faces? _________

Does this contradict your conjecture above? Explain. Hint: carefully consider the definitions above.