

Friday, 9/4 - ①

More on mathematical induction.

Theorem.

$$\forall n \in \mathbb{N}, \\ 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

Proof.

Let $A(n)$ be the above statement.

Step 1: Is $A(1)$ true?

$$1^2 \stackrel{?}{=} \frac{1(1+1)(2 \cdot 1 + 1)}{6}$$

$$1 = 1. \quad \checkmark$$

So $A(1)$ is true.

[right side of $A(k+1)$
is $(k+1)(k+2)(2k+3)/6$]

Step 2: Assume

$$A(k): 1^2 + 2^2 + 3^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6}.$$

Then

$$\begin{aligned} & 1^2 + 2^2 + \dots + (k+1)^2 \\ &= (1^2 + 2^2 + \dots + k^2) + (k+1)^2 \\ &= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \\ &= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6} \\ &= \frac{(k+1)(k(2k+1) + 6(k+1))}{6} \\ &= \frac{(k+1)(2k^2 + k + 6k + 6)}{2} \end{aligned}$$

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$$= \underline{(k+1)(k+2)(2k+3)}$$

$$= \underline{(k+1) \overset{6}{((k+1)+1)} (2(k+1)+3))}.$$

So $A(k+1)$ follows.

So $A(k) \Rightarrow A(k+1)$.

By induction, $A(n)$ is true $\forall n \in \mathbb{N}$.

□