Monday 8/25-(1
Statements and Proofs.
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T TI 11. + D->A
I. The statement $P \Rightarrow Q$. Read "if P then Q;" "P implies Q." Meaning: whenever P is true, Q must follow.
Read if 4 then Q; Pimples Q.
Meaning: whenever P is true, Q must follow.
\sim
Methods of proof:
A) And amof Such a amof looks like this:
A) Direct proof. Such a proof looks like this?
0 1 0 5 0
Proof. Proof.
Proof.
Assume P. [Now do what's necessary to
conclude: 7 Therefore Q.
Proof. Assume P. [Now do what's necessary to conclude:] Therefore, Q. So P => Q.
1_ direct proof
1_ direct proof template"
V
Example 1:
Proposition 1. If n is on odd integer, then n=1 is divisible by 4. Proof
Proposition 1.
If n is on only integer, then n=1 is divisible
b. 4
p d .f
Proof Assume n is an odd integer: then $n=2k+1$ for some integer k. So
Assume his an odd integer. Then hearth
tor some integer k. So
$n^{2}-1=(2k+1)^{2}-1=4k^{2}+4k+1-1=4k^{2}+4k$
$n^{2}-1=(2k+1)^{2}-1=4k^{2}+4k+1-1=4k^{2}+4k$ =4(k^{2}+k).
So ha 1 is divisible by 4
andes"
So n^2-1 is alwisible by 4. So $n \in \mathbb{Z}$ is odd => 4 (n^2-1) .
So $n \in \mathbb{Z}_{i}$ is odd => 4 (n°-1).
* 41 4 -4 w4 = - ~

"not P" (the Q)
B) Contrapositive proof. "not P" (the 2) negation of P)
FACT: the statement P=>Q is always
B) Contrapositive proof. FACT: the statement $P = > Q$ is always logically equivalent to its contrapositive $\sim Q = > \sim P$.
So a contrapositive proof looks like: contrapositive proof "templote"
Proposition. $P \Rightarrow Q$.
Proof.
Assume ~ Q. I Then do what's necessary to conclude: 1 Therefore, ~ P.
conclude I therefore, or.
So $P = > Q$.
Example 2:
Proposition 2.
$n \in \mathbb{Z}$ is odd => $4/(n^2-1)$.
ne Z is odd => $4/(n^2-1)$. Proof. Assume $4/(n^2-1)$.
Assume 47 (n21).
wntc
n=2k+r where r=0 (if n is even) or r=1
(if n is odd). Then $n^{2}-1=(2k+r)^{2}-1=4k^{2}+4kr+r^{2}-1$
$n^{-1} = (dk+r)^{-1} = 4k^{+} + 4kr + r - 1$ = $4(k^{2} + kr) + (r^{2} - 1)$.
Since $41(n^2-1)$, we see that $r^2-1 \neq 0$, so $r \neq 1$, so $r = 0$. So n is even (and therefore not odd).
So $n \in \mathbb{Z}$ is all $\Rightarrow 4/(n^2-1)$.
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Il Proof by contradiction.
The idea: to prove a statement T, assume
~ 1. Show this leads to an absorbity. Then ~ 1
The idea: to prove a statement T, assume ~T. Show this leads to an absurdity. Then ~T must be false, so T is true.
*often, this will be of the form "Vand 2V," for some other statement V.

Like this:

Proposition. T.		
Proof		
Assume ~ T. [Then do stuff to conclude:]		
Therefore, V. [Then do m	ore stuff to conclude:	
Therefore, ~ V. Contradic		
•		
Therefore, T.		
Contradiction	proof "template".	
Example 3:	(1) definition there are analysis	
Paraston 3.	(by definition, these are positive	
There are in finitely my	The John a Mumbers.	
THE CAIC TRITING TO	the document T	
Proposition 3. There are infinitely me (this is ~T)		
Proof. Assume there are	only finitely many prime	
Proof. Assume there are numbers:		
call them pa, pz, pz,	-, pk.	
and let p be any prime dr	pk+1,	
and let p be any prime dr	disor of M.	
(G) The only primes are parp	z,, pk, so p equalsone	
(G) The only primes are popposed of these primes, so p div	ides their product is	
$\mathcal{N} = \mathcal{D}_1 \mathcal{D}_2 \cdots \mathcal{D}_k$	V, p divides their difference,	
Since p divides Mand A	V. D divides their difference	
50 pl(M-N).		