

# Conway's Soldiers

Please answer all questions in complete sentences.

Conway's Soldiers is a one-person mathematical game or puzzle devised and analyzed by mathematician John Horton Conway in 1961. A variant of peg solitaire, it takes place on an infinite checkerboard. The board is divided by a horizontal line that extends indefinitely. Above the line are empty cells and below the line are an arbitrary number of game pieces, or "soldiers." As in peg solitaire, a move consists of one soldier jumping over an adjacent soldier into an empty cell, vertically or horizontally (but not diagonally), and removing the soldier which was jumped over. The goal of the puzzle is to place a soldier as far above the horizontal line as possible.

1. Come up with a strategy that will allow a soldier to advance to the second level (second row above the horizontal line). You may draw pictures to depict your strategy.

2. What is the highest level that can be reached by a soldier? What strategy did you use to reach this level? You may draw pictures to depict your strategy.

3. If diagonal jumps are allowed, can you find a strategy that will allow a soldier to advance farther than you previously could? Come up with a strategy and explain. [**Hint:** Try working backwards.]

Chomp is a two-player strategy game played on a rectangular chocolate bar made up of smaller square blocks (cells). The players take it in turns to choose one block and “eat it” (remove from the board), together with those that are below it and to its right. The top left block is “poisoned” and the player who eats this loses.

4. Play the game several times with various sizes for the rectangular chocolate bar. Explain why there is always a strategy so that the first player wins (unless the starting bar is  $1 \times 1$ ).

Chopsticks is a hand game for two players, in which players extend a number of fingers from each hand and transfer those scores by taking turns to tap one hand against another. The rules are simple:

- Each player uses both hands to play the game, the number of fingers extended on a hand shows the number of points that the hand has.
  - Both players start with each hand having one point – one finger extended on each hand.
  - The goal of the game is for a player to force their opponent to extend all of their fingers and thumbs on both hands or to force the player to extend all their fingers and thumb and one hand if their other hand is already out.
  - A hand with all fingers and its thumb extended is called a dead hand and is taken out of play.
  - Players take turns to tap their live hand(s) against another live hand (either their own other hand, or one of their opponent's). You can tap any live hand per turn but only one.
  - The number of points on the tapping hand is added to the number on the tapped hand, and the player with the tapped hand extends their digits to show the new score. The tapping hand remains unchanged.
  - A player may tap their two hands together to transfer points from one hand to the other. This is called a split. For example, if a player has 4 on one hand and 2 on the other, they could split to have 3 on each hand. Additionally, if a player has one hand out (0 fingers) and two fingers on the other hand, they could split to have one finger on each hand. You are not allowed to “swap hands” or switch the number of fingers between hands without splitting (going from 4 on one hand and 2 on the other to 2 on one hand and 4 on the other).
5. Play the game several times. Determine a winning strategy so that the first player always wins regardless of what the second player does.