Select solutions to Homework #1

3.8 Suppose $f(x_1) = f(x_2)$. Then, by definition of f, $3x_1 - 5 = 3x_2 - 5$. Or, in other words, $3x_1 = 3x_2$, and thus $x_1 = x_2$. This verifies the contrapositive implication and hence the original implication as well.

4.16 Assume $\sqrt{x}$ is rational. Then there exists integers p, q such that $\sqrt{x} = p/q$. Thus, $x = p^2/q^2$. Since p^2, q^2 are integers, it follows that x is also rational. This verifies the contrapositive implication and hence the original implication as well.

4.25 Let $x > 0$. As $2x > 0$, we have

$$x^2 + 1 < x^2 + 2x + 1 = (x+1)^2.$$

Observe that $2x \leq x^2 + 1$; this follows from the trivial bound $x^2 - 2x + 1 = (x-1)^2 \geq 0$. Therefore, we obtain

$$(x+1)^2 = x^2 + 2x + 1 \leq 2x^2 + 2,$$

and the proof of both inequalities is complete.

7.3 (a) $[2, \infty)$

(b) $[-3, \infty)$

(c) Since $x^2 + 6x + 4 = (x+3)^2 - 5$, the range of f is $[-5, \infty)$.

(d) $[-3, 3]$