

Final Exam

Linear Algebra: Matrix Methods

MATH 2130

Fall 2025

Tuesday December 9, 2025

UPLOAD THIS COVER SHEET!

NAME: _____

PRACTICE EXAM

Question:	1	2	3	4	5	6	Total
Points:	20	20	20	20	10	10	100
Score:							

- The exam is closed book. You **may not use any resources** whatsoever, other than paper, pencil, and pen, to complete this exam.
- You **may not discuss the exam** with anyone except me, in any way, under any circumstances.
- You **must explain your answers**, and you will be **graded on the clarity of your solutions**.
- You must upload your exam as a single **.pdf** to **Canvas**, with the questions in the correct order, etc.
- You have 70 minutes to complete the exam.

1. (20 points) • Let $\mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{x}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, and $\mathbf{x}_4 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$.

Use the Gram–Schmidt process to find an orthonormal basis for the vector subspace of \mathbb{R}^4 spanned by the vectors \mathbf{x}_1 , \mathbf{x}_2 , \mathbf{x}_3 , and \mathbf{x}_4 .

Total for Question 1: 20



2. (20 points) • For the vectors \mathbf{x}_1 , \mathbf{x}_2 , \mathbf{x}_3 , and \mathbf{x}_4 of the previous problem, find the vector in the span of

those vectors that is closest to the vector $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

Total for Question 2: 20



3. (20 points) • Find the equation $y = \beta_0 + \beta_1 x$ of the line that best fits the given data points, as a least squares model:

$$\begin{bmatrix} x \\ y \end{bmatrix} : \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Total for Question 3: 20



4. • Consider the following real matrix

$$A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$$

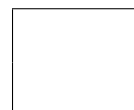
- (a) (5 points) Find the characteristic polynomial $p_A(t)$ of A .

(b) (5 points) *Find the eigenvalues of A .*

(c) (5 points) *Find a basis for each eigenspace of A in \mathbb{R}^3 .*

(d) (5 points) *Is A diagonalizable? If so, find a matrix $S \in M_{3 \times 3}(\mathbb{R})$ so that $S^{-1}AS$ is diagonal. If not, explain.*

Total for Question 4: 20



5. • Consider the 2-dimensional discrete dynamical system

$$\mathbf{x}_{k+1} = A\mathbf{x}_k$$

where

$$A = \begin{pmatrix} 1.7 & 0.3 \\ 1.2 & 0.8 \end{pmatrix}$$

(a) (5 points) *Is the origin an attractor, repeller, or saddle point?*

(b) (5 points) *Find the directions of greatest attraction or repulsion.*

Total for Question 5: 10



6. • **TRUE** or **FALSE**. For this problem, and this problem only, **you do not need to justify your answer.**

(a) (2 points) **TRUE** or **FALSE** (circle one). If $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, then $|\mathbf{x} \cdot \mathbf{y}| \leq \|\mathbf{x}\| \|\mathbf{y}\|$.

(b) (2 points) **TRUE** or **FALSE** (circle one). Two vectors in \mathbb{R}^n are orthogonal if their dot product is zero.

(c) (2 points) **TRUE** or **FALSE** (circle one). If $A \in M_{m \times n}(\mathbb{R})$ and $\mathbf{b} \in \mathbb{R}^m$, then a least squares solution to the equation $A\mathbf{x} = \mathbf{b}$ is a vector $\hat{\mathbf{x}} \in \mathbb{R}^n$ such that $A^T A \hat{\mathbf{x}} = A^T \mathbf{b}$.

(d) (2 points) **TRUE** or **FALSE** (circle one). If A is any real matrix, then the matrix $A^T A$ has non-negative eigenvalues.

(e) (2 points) **TRUE** or **FALSE** (circle one). Given symmetric matrices A and B of the same size, i.e., $A = A^T$ and $B = B^T$, then AB is a symmetric matrix, i.e., $AB = (AB)^T$.

Total for Question 6: 10