# Final Exam

## Linear Algebra: Matrix Methods

## MATH 2130

## Fall 2025

Tuesday December 9, 2025

**UPLOAD THIS COVER SHEET!**

NAME: ______________________________

PRACTICE EXAM

| Question: | **1** | **2** | **3** | **4** | **5** | **6** | Total |
|---|---|---|---|---|---|---|---|
| Points: | 20 | 20 | 20 | 20 | 10 | 10 | 100 |
| Score: | | | | | | | |

- The exam is closed book. You **may not use any resources** whatsoever, other than paper, pencil, and pen, to complete this exam.
- You **may not discuss the exam** with anyone except me, in any way, under any circumstances.
- You **must explain your answers**, and you will be **graded on the clarity of your solutions**.
- You must upload your exam as a single **.pdf** to **Canvas**, with the questions in the correct order, etc.
- You have 70 minutes to complete the exam.

**1.** (20 points) • Let $\mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{x}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, and $\mathbf{x}_4 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$.

*Use the Gram–Schmidt process to find an orthonormal basis for the vector subspace of* $\mathbb{R}^4$ *spanned by the vectors* $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$, *and* $\mathbf{x}_4$.

Total for Question **1: 20**

**2.** (20 points) • For the vectors $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$, and $\mathbf{x}_4$ of the previous problem, find the vector in the span of those vectors that is closest to the vector $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

Total for Question **2: 20**

**3.** (20 points) • *Find the equation $y = \beta_0 + \beta_1 x$ of the line that best fits the given data points, as a least squares model:*

$$\begin{bmatrix} x \\ y \end{bmatrix} : \quad \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Total for Question **3: 20**

**4.** • Consider the following real matrix

$$A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$$

(a) (5 points) *Find the characteristic polynomial* $p_A(t)$ *of* $A$.

(b) (5 points) *Find the eigenvalues of $A$.*

(c) (5 points) *Find a basis for each eigenspace of $A$ in $\mathbb{R}^3$.*

(d) (5 points) *Is $A$ diagonalizable? If so, find a matrix $S \in \mathrm{M}_{3\times 3}(\mathbb{R})$ so that $S^{-1}AS$ is diagonal. If not, explain.*

Total for Question **4: 20**

**5.** • Consider the 2-dimensional discrete dynamical system

$$\mathbf{x}_{k+1} = A\mathbf{x}_k$$

where

$$A = \begin{pmatrix} 1.7 & 0.3 \\ 1.2 & 0.8 \end{pmatrix}$$

(a) (5 points) *Is the origin an attractor, repeller, or saddle point?*

(b) (5 points) *Find the directions of greatest attraction or repulsion.*

Total for Question **5: 10**

**6.** • **TRUE** or **FALSE**. For this problem, and this problem only, **you do not need to justify your answer**.

(a) (2 points) **TRUE** or **FALSE** (circle one). *If* $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, *then* $|\mathbf{x}.\mathbf{y}| \leq \|\mathbf{x}\|\|\mathbf{y}\|$.

(b) (2 points) **TRUE** or **FALSE** (circle one). *Two vectors in* $\mathbb{R}^n$ *are orthogonal if their dot product is zero.*

(c) (2 points) **TRUE** or **FALSE** (circle one). *If* $A \in \mathrm{M}_{m\times n}(\mathbb{R})$ *and* $\mathbf{b} \in \mathbb{R}^m$, *then a least squares solution to the equation* $A\mathbf{x} = \mathbf{b}$ *is a vector* $\hat{\mathbf{x}} \in \mathbb{R}^n$ *such that* $A^T A\hat{\mathbf{x}} = A^T\mathbf{b}$.

(d) (2 points) **TRUE** or **FALSE** (circle one). *If* $A$ *is any real matrix, then the matrix* $A^T A$ *has non-negative eigenvalues.*

(e) (2 points) **TRUE** or **FALSE** (circle one). *Given symmetric matrices* $A$ *and* $B$ *of the same size, i.e.,* $A = A^T$ *and* $B = B^T$, *then* $AB$ *is a symmetric matrix, i.e.,* $AB = (AB)^T$.

Total for Question **6: 10**