

Exercise 4.1.6

Linear Algebra

MATH 2130

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 4.1.6 from Lay [LLM21, §4.1]:

Exercise 4.1.6. Determine if the set of all polynomials

$$\{p(t) = a + t^2 : a \in \mathbb{R}\}$$

is a subspace of $\mathbb{R}[t]_n$ (“ $\mathbb{P}_n$ ”), the vector space of real polynomials in t of degree at most n , for an appropriate value of n .

Solution. The set of all polynomials

$$\{p(t) = a + t^2 : a \in \mathbb{R}\}$$

is *not* a subspace of $\mathbb{R}[t]_n$ (“ $\mathbb{P}_n$ ”) for any n . It is not even a subspace of $\mathbb{R}[t]$, the space of all polynomials.

Indeed, the set is not closed under addition. If we have $a + t^2$, and we add it to itself, we get $(a + t^2) + (a + t^2) = 2a + 2t^2$, which is not in the given set of polynomials since the coefficient of t^2 is not equal to 1. (Note that the observation that $2(a + t^2) = 2a + 2t^2$ also shows the set is not closed under scaling.) □

REFERENCES

[LLM21] David Lay, Stephen Lay, and Judi McDonald, *Linear Algebra and its Applications*, Sixth edition, Pearson, 2021.

UNIVERSITY OF COLORADO, DEPARTMENT OF MATHEMATICS, CAMPUS BOX 395, BOULDER, CO 80309

Email address: `casa@math.colorado.edu`