

Exercise 3.9.6

Introduction to Discrete Mathematics MATH 2001

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ABSTRACT. This is Exercise 3.9.6 from Hammack [[Ham13](#), §3.9]:

Exercise 3.9.6. Given a sphere S , a *great circle* of S is the intersection of S with a plane through its center. Every great circle divides S into two parts. A *hemisphere* is the union of the great circle and one of these two parts. Show that if five points are placed arbitrarily on S , then there is a hemisphere that contains four of them.

Solution. Place the first two of the five points on the sphere. These two points and the center of the sphere define a plane, and therefore, these first two points lie on a great circle. Now consider the two hemispheres determined by this great circle. The Division Principle states that 2 of the remaining 3 points placed on the circle must be in one of the two hemispheres. Therefore, those 2 points, as well as the 2 points used to define the great circle, lie in the same hemisphere. \square

REFERENCES

[Ham13] Richard Hammack, *Book of proof*, Creative Commons, 2013.

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