

Exercise 12.6.6

Introduction to Discrete Mathematics MATH 2001

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ABSTRACT. This is Exercise 12.6.6 from Hammack [Ham13, §12.6]:

Exercise 12.6.6. True or False: If $f : A \rightarrow B$ is a map of sets (“function”) and $Y \subseteq B$, then $f(f^{-1}(Y)) = Y$. Prove this statement, or give a counterexample.

Solution. This statement is FALSE. Indeed, let $A = \{1\}$, $B = \{1, 2\}$, let $f : A \rightarrow B$ be the map given by the rule $f(1) = 1$, and let $Y = B$. Then $f(f^{-1}(Y)) = f(A) = \{1\} \subsetneq \{1, 2\} = Y$. \square

Remark 0.1. Note that if $f : A \rightarrow B$ is a map of sets (“function”) and $Y \subseteq B$, then $f(f^{-1}(Y)) \subseteq Y$. Indeed, we have that $f^{-1}(Y) = \{a \in A : f(a) \in Y\}$, so that if $a \in f^{-1}(Y)$, we have $f(a) \in Y$, and therefore, $f(f^{-1}(Y)) \subseteq Y$.

REFERENCES

[Ham13] Richard Hammack, *Book of proof*, Creative Commons, 2013.

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