Exercise 12.6.5

Introduction to Discrete Mathematics
MATH 2001

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ABSTRACT. This is Exercise 12.6.5 from Hammack [Ham13, §12.6]:

Exercise 12.6.5. Given a map of sets ("function") $f : A \to B$ and a subset $X \subseteq A$, prove that $X \subseteq f^{-1}(f(X))$.

Remark 0.1. Note that we observed in [Ham13, Example 12.14] that we may have $X \neq f^{-1}(f(X))$.

Solution. We have by definition that

$$f^{-1}(f(X)) = \{a \in A : f(a) \in f(X)\} \quad \text{and} \quad f(X) = \{f(x) : x \in X\}.$$

Consequently, since for all $x \in X$ we have $f(x) \in f(X)$, we have that $x \in f^{-1}(f(X))$, so that $X \subseteq f^{-1}(f(X))$. □

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REFERENCES

[Ham13] Richard Hammack, *Book of proof*, Creative Commons, 2013.

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