Exercise 12.3.2

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ABSTRACT. This is Exercise 12.3.2 from Hammack [Ham13, §12.3]:

Exercise 12.3.2. Prove that if a is a natural number, then there exist two unequal natural numbers k and ℓ for which $ak - a\ell$ is divisible by 10.

Solution. Consider the map $f : \{0, 1, 2, \ldots, 11\} \to \mathbb{Z}_{10}$ defined by $f(x) = ax \pmod{10}$. Since $|\{0, 1, 2, \ldots, 10\}| = 11 > 10 = |\mathbb{Z}_{10}|$, the Pigeonhole Principle implies that f is not injective. So there exist some $y \in \mathbb{Z}_{10}$ such that there exists $k, \ell \in \{0, 1, 2, \ldots, 10\}$ with $k \neq \ell$ such that $ak \pmod{10} = f(k) = y = f(\ell) = a\ell \pmod{10}$. Therefore, $ak - a\ell = 0 \pmod{10}$, so that $ak - a\ell$ is divisible by 10. $\square$

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References

[Ham13] Richard Hammack, *Book of proof*, Creative Commons, 2013.

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