

Take-Home Final

Abstract Algebra 1

MATH 3140

Summer 2021

Friday July 2, 2021

NAME: _____

PRACTICE EXAM

Question:	1	2	3	4	Total
Points:	25	25	25	25	100
Score:					

- For the exam you may use **only the following resources** from this course: our textbook, your lecture notes, my lecture notes, your homework, the pdfs linked from the course webpage:
<http://math.colorado.edu/~casa/teaching/21summer/3140/hw.html>
and the quizzes and midterms we have taken on Canvas.
- You **may not use any other resources** whatsoever.
- You **may not discuss the exam** with anyone except me, in any way, under any circumstances.
- You **must explain your answers**, and you will be **graded on the clarity of your solutions**.
- You must upload your solutions to **Canvas** as a **single .pdf** file with the questions in the correct order.
- The exam is due at 10:00 PM Friday July 2, 2021.

1. (25 points) • Let G be a group with center $Z(G)$. Show that if $G/Z(G)$ is cyclic, then $Z(G) = G$. [Hint: Show first there exists $g \in G$ such that for any $g_1 \in G$, there is a $z_1 \in Z(G)$ and $n_1 \in \mathbb{Z}$ such that $g_1 = g^{n_1}z_1$. Then show for any $g_1, g_2 \in G$ that $g_1g_2 = g_2g_1$.]

1
25 points

2. (a) (15 points) • In a commutative ring with unity, show that $(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}$.

(b) (10 points) An element r of a ring R is said to be nilpotent if there exists some $n \in \mathbb{N}$ such that $r^n = 0$. Let N be the set of nilpotent elements of a commutative ring R with unity. Show that N is an ideal in R .

2
25 points

3. (25 points) • Let D be an integral domain, and suppose that for every descending chain of ideals in D

$$\cdots \subseteq I_4 \subseteq I_3 \subseteq I_2 \subseteq I_1 \subseteq D$$

there is a positive integer n such that $I_m = I_n$ for all $m \geq n$. Show that D is a field.

3
25 points

4. (25 points) • Show that if F , E , and K are fields with $F \leq E \leq K$, then K is algebraic over F if and only if K is algebraic over E , and E is algebraic over F . (You must *not* assume the extensions are finite.)

4
25 points