In-Class Final

Abstract Algebra 1

MATH 3140

Summer 2021

Friday July 2, 2021

NAME: ______________________________

PRACTICE EXAM

Question:	**1**	**2**	**3**	**4**	**5**	Total
Points:	20	20	20	20	20	100
Score:						

- You must have your **camera** on, and a working **microphone**, for the **duration of the exam** in order to receive credit.
- The exam is closed book. You **may not use any resources** whatsoever, other than paper, pencil, and pen, to complete this exam.
- You **may not discuss the exam** with anyone except me, in any way, under any circumstances.
- You **must explain your answers**, and you will be **graded on the clarity of your solutions**.
- You must upload your exam as a single **.pdf** to **Canvas**, with the questions in the correct order, etc.
- You have 60 minutes to complete the exam.

1. • Consider the dihedral group D_n, with $n \geq 3$. Recall the notation we have been using: D_n has identity element I, and is generated by elements R and D, satisfying the relations $R^n = D^2 = I$ and $RD = DR^{-1}$. Consider the cyclic subgroup $\langle R^2 \rangle$.

(a) (10 points) *Show that* $\langle R^2 \rangle$ *is a normal subgroup of* D_n.

(b) (10 points) *Find the order of the group* $D_n / \langle R^2 \rangle$. [*Hint:* this may depend on the parity of n.]

1
20 points

2. • Consider the map (or "function") of polynomial rings

$$\phi : \mathbb{Z}[x] \longrightarrow \mathbb{Z}_4[x]$$

$$\sum_{k=0}^{n} a_k x^k \mapsto \sum_{k=0}^{n} [a_k] x^k,$$

where $[a_k] = a_k \pmod 4$.

(a) (10 points) *Show that ϕ is a homomorphism of rings.*

(b) (10 points) *Describe the kernel of ϕ.* (Do not just write down the definition; you need to describe an explicit subset of $\mathbb{Z}[x]$.)

2
20 points

3. (20 points) • *Show that for a prime p, the polynomial $x^p + a \in \mathbb{Z}_p[x]$ is not irreducible for any $a \in \mathbb{Z}_p$.*

3
20 points

4. (20 points) • *Prove that the algebraic closure of* $\mathbb{Q}$ *in* $\mathbb{C}$ *is not a finite extension of* $\mathbb{Q}$.

4
20 points

5. (20 points) • Find the degree and a basis for the field extension $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ over $\mathbb{Q}$.

5
20 points