

Exercise 7.2

Abstract Algebra 1

MATH 3140

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 7.2 from Fraleigh [Fra03, §7]:

Exercise 7.2. List the elements of the subgroup of \mathbb{Z}_{12} generated by the subset $\{4, 6\}$.

Solution. The subgroup of \mathbb{Z}_{12} generated by $\{4, 6\}$ is the subgroup $\{0, 2, 4, 6, 8, 10\}$.

Indeed, since $\gcd(4, 6) = 2$, the subgroup of \mathbb{Z}_{12} generated by the subset $\{4, 6\}$ is the same as the subgroup generated by 2 (or more directly, since $2 = 6 - 4$, the subgroup generated by 2 is contained in the subgroup generated by $\{4, 6\}$, and since 4 and 6 are in the subgroup generated by 2, the subgroup generated by 2 is equal to the subgroup generated by $\{4, 6\}$). The subgroup of \mathbb{Z}_{12} generated by 2 is the subgroup $\{0, 2, 4, 6, 8, 10\}$. \square

REFERENCES

[Fra03] John Fraleigh, *A First Course in Abstract Algebra*, Seventh edition, Addison Wesley, Pearson, 2003.

UNIVERSITY OF COLORADO, DEPARTMENT OF MATHEMATICS, CAMPUS BOX 395, BOULDER, CO 80309

Email address: casa@math.colorado.edu