Exercise 4.35

Abstract Algebra 1
MATH 3140

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 4.35 from Fraleigh [Fra03, §4]:

Exercise 4.35. Show that if $(a * b)^2 = a^2 * b^2$ for a and b in a group G, then $a * b = b * a$.

Solution. Suppose that $(a * b)^2 = a^2 * b^2$ for a and b in a group G. Then we have

$$(a * b) * (a * b) = (a * a) * (b * b).$$

We can multiply both sides by a^{-1} on the left, and b^{-1} on the right, giving $a^{-1} * a * b * a * b = a^{-1} * a * b * b * b^{-1}$, which reduces to $e * b * a * e = e * a * b * e$, which is the same as $b * a = a * b$. $\square$

REFERENCES

[Fra03] John Fraleigh, *A First Course in Abstract Algebra*, Seventh edition, Addison Wesley, Pearson, 2003.

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