Exercise 4.34

Abstract Algebra 1
MATH 3140

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 4.34 from Fraleigh [Fra03, §4]:

Exercise 4.34. Let G be a group with a finite number of elements. Show that for any $a \in G$, there exists an $n \in \mathbb{Z}^+$ such that $a^n = e$. [*Hint:* Consider $e, a, a^2, \ldots, a^m$, where m is the number of elements in G, and use the cancellation laws.]

Solution. Let $a \in G$, and consider the list $e, a, a^2, \ldots, a^m$, where m is the number of elements in G. Since there are only m elements in G, and there are $m + 1$ elements in the list, it must be that there are two elements in the list that are equal. In other words, there must be $r, s \in \{0, 1, \ldots, m\}$ with $r < s$ such that $a^r = a^s$. Then multiplying on the right by a^{-r} $(:= (a^{-1})^r)$, we have that $e = a^{s-r}$. Setting $n = s - r > 0$, we have that $a^n = e$, as desired. $\square$

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REFERENCES

[Fra03] John Fraleigh, *A First Course in Abstract Algebra*, Seventh edition, Addison Wesley, Pearson, 2003.

UNIVERSITY OF COLORADO, DEPARTMENT OF MATHEMATICS, CAMPUS BOX 395, BOULDER, CO 80309

Email address: `casa@math.colorado.edu`