Exercise 22.24

Abstract Algebra 1
MATH 3140

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 22.24 from Fraleigh [Fra03, §22]:

Exercise 22.24. Prove that if D is an integral domain, then $D[x]$ is an integral domain.

Solution. If D is an integral domain, then $D[x]$ is a commutative ring with unity $1 \neq 0$, and so to prove that $D[x]$ is an integral domain, we must show that $D[x]$ has no divisors of 0.

In other words, if $0 \neq f(x), g(x) \in D[x]$, we must show that $f(x)g(x) \neq 0$. To this end, if $f(x) \neq 0$, then we may write $f(x) = a_0 + a_1x + \cdots + a_dx^d \in D[x]$ with $a_d \neq 0$, and similarly, if $g(x) \neq 0$, then we may write $g(x) = b_0 + b_1x + \cdots + b_ex^e \in D[x]$ with $b_e \neq 0$. We then have

$$\begin{aligned} f(x)g(x) &= (a_0 + a_1x + \cdots + a_dx^d)(b_0 + b_1x + \cdots + b_ex^e) \\ &= a_0b_0 + (a_0b_1 + a_1b_0)x + \cdots + a_db_ex^{d+e}. \end{aligned}$$

Now since a_d and b_d are nonzero, and D is an integral domain, we have that $a_db_e \neq 0$, so that $f(x)g(x) \neq 0$. Thus $D[x]$ is an integral domain. □

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REFERENCES

[Fra03] John Fraleigh, *A First Course in Abstract Algebra*, Seventh edition, Addison Wesley, Pearson, 2003.

UNIVERSITY OF COLORADO, DEPARTMENT OF MATHEMATICS, CAMPUS BOX 395, BOULDER, CO 80309

Email address: casa@math.colorado.edu