Exercise 13.2

Abstract Algebra 1
MATH 3140

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 13.2 from Fraleigh [Fra03, §13]:

Exercise 13.2. Determine whether the map $\phi : \mathbb{R} \to \mathbb{Z}$ given by $\phi(x) = \lfloor x \rfloor$ (i.e., the greatest integer that is less than or equal to x) is a homomorphism of groups.

Solution. The map $\phi : \mathbb{R} \to \mathbb{Z}$ given by $\phi(x) = \lfloor x \rfloor$ is *not* a homomorphism of groups. For instance, we have

$$\phi\left(\frac{1}{2} + \frac{1}{2}\right) = \phi(1) = \lfloor 1 \rfloor = 1 \neq 0 = 0 + 0 = \lfloor \frac{1}{2} \rfloor + \lfloor \frac{1}{2} \rfloor = \phi\left(\frac{1}{2}\right) + \phi\left(\frac{1}{2}\right).$$

□

Remark 0.1. Another way to do this problem would be to observe that $\phi^{-1}(0) = [0, 1) \subseteq \mathbb{R}$ is not a subgroup (since for instance $\frac{1}{2} \in [0, 1)$, but $\frac{1}{2} + \frac{1}{2} \notin [0, 1)$).

Date: August 8, 2021.

REFERENCES

[Fra03] John Fraleigh, *A First Course in Abstract Algebra*, Seventh edition, Addison Wesley, Pearson, 2003.

UNIVERSITY OF COLORADO, DEPARTMENT OF MATHEMATICS, CAMPUS BOX 395, BOULDER, CO 80309

Email address: `casa@math.colorado.edu`