Exercise 10.3

Abstract Algebra 1
MATH 3140

SEBASTIAN CASALAINA

ABSTRACT. This is Exercise 10.3 from Fraleigh [Fra03, §10]:

Exercise 10.3. Find all cosets of the subgroup $\langle 2 \rangle$ of $\mathbb{Z}_{12}$.

Solution. There are two cosets of the subgroup $\langle 2 \rangle$ of $\mathbb{Z}_{12}$, namely,

$$\langle 2 \rangle \quad \text{and} \quad 1 + \langle 2 \rangle.$$

To see this, observe that

$$0 + \langle 2 \rangle = \langle 2 \rangle = \{0, 2, 4, 6, 8, 10\}.$$
$$1 + \langle 2 \rangle = \{1, 3, 5, 7, 9, 11\}.$$

These left cosets are distinct. By Lagrange's theorem, the number of left cosets of $\langle 2 \rangle$ in $\mathbb{Z}_{12}$ is given by:

$$(\mathbb{Z}_{12} : \langle 2 \rangle) = |\mathbb{Z}_{12}|/|\langle 2 \rangle| = 12/6 = 2,$$

so $\langle 2 \rangle$ and $1 + \langle 2 \rangle$ are the left only cosets. Since the group $\mathbb{Z}_{12}$ is abelian, every left coset is also a right coset, and so these are all the cosets. □

Remark 0.1. Alternatively, we can conclude that $0 + \langle 2 \rangle$ and $1 + \langle 2 \rangle$ are the only two cosets by observing that every element of $\mathbb{Z}_{12}$ is contained in one of these two left cosets, and again, since $\mathbb{Z}_{12}$ is abelian, every left coset is also a right coset.

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REFERENCES

[Fra03] John Fraleigh, *A First Course in Abstract Algebra*, Seventh edition, Addison Wesley, Pearson, 2003.

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