Midterm 1

Linear Algebra

MATH 2130

Spring 2021

Friday February 12, 2021

NAME: **Enter your name here** ______________________________

PRACTICE EXAM

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	60	

- This exam is closed book.
- You may use only paper and pencil.
- You may not use any other resources whatsoever.
- You will be graded on the clarity of your exposition.

1. (10 points) • **TRUE** or **FALSE**: *Suppose that* $V \subseteq \mathbb{R}^n$ *is a nonempty subset satisfying:*

1. *For all* $v_1, v_2 \in V$*, we have* $v_1 + v_2 \in V$.
2. *For all* $v \in V$*, we have* $-v \in V$.

Then V *is a subspace of* $\mathbb{R}^n$.

If true, state this clearly at the start of your solution, and provide a proof. If false, state this clearly at the start of your solution, provide a counterexample, and prove that it is a counterexample.

2. (10 points) • *Find all solutions to the following system of linear equations:*

$$\begin{array}{rcrcrcr} 3x_1 & + & 9x_2 & + & 27x_3 & = & -3 \\ -3x_1 & - & 11x_2 & - & 35x_3 & = & 5 \\ 2x_1 & + & 8x_2 & + & 26x_3 & = & -4 \end{array}$$

3. (10 points) • Consider the matrix

$$A = \begin{bmatrix} 1 & -3 & 0 & -1 & 4 & -2 \\ 0 & 0 & 1 & -1 & 0 & 1 \\ 3 & -9 & 0 & -3 & 2 & 4 \\ 1 & -3 & 1 & -2 & 4 & -1 \end{bmatrix}$$

(a) *Find the reduced row echelon form of A.*

(b) *Are the columns of A linearly independent?*

(c) *Are the rows of A linearly independent?*

(d) *What is the column rank of A?*

(e) *What is the row rank of A?*

4. (10 points) • Consider the linear map $L : \mathbb{R}^3 \to \mathbb{R}^2$ given by

$$L(x_1, x_2, x_3) = (2x_1 - x_3, 3x_2 + x_3).$$

Write down the matrix form of the linear map L.

5. (10 points) • Consider the matrix

$$B = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

(a) *Find the inverse of B.*

(b) *Does there exist $x \in \mathbb{R}^3$ such that* $Bx = \begin{bmatrix} 5 \\ \sqrt{2} \\ \pi \end{bmatrix}$?

6. (10 points) • **TRUE** or **FALSE**:

(a) Let $A \in M_{m \times n}(\mathbb{R})$. There is an $x \in \mathbb{R}^n$ such that $Ax = 0$.

(b) Let $A \in M_{m \times n}(\mathbb{R})$. If the columns of A span $\mathbb{R}^m$, then for any $b \in \mathbb{R}^m$ there is an $x \in \mathbb{R}^n$ such that $Ax = b$.

(c) The map $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$ for all $x \in \mathbb{R}$ is a linear map.

(d) If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then $A^n = \begin{bmatrix} 1 & 2^{n-1} \\ 0 & 1 \end{bmatrix}$ for each natural number n.

(e) If A and B are $m \times n$ matrices, then $A + B = B + A$.

(f) Let $A \in M_{m \times n}(\mathbb{R})$. If the rows of A are linearly independent, then for any $b \in \mathbb{R}^m$ there is at most one $x \in \mathbb{R}^n$ such that $Ax = b$.

(g) Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be a linear map. The kernel of f is a sub-vector space of $\mathbb{R}^n$.

(h) If the columns of a square matrix A are linearly independent, then A^T is invertible.

(i) If $V, W \subseteq \mathbb{R}^n$ are subspaces. The union $V \cup W$ is a subspace of $\mathbb{R}^n$.

(j) Suppose that A and B are square matrices, and AB is invertible. Then A and B are invertible.