Proof of Concept

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For math majors

How to do your homework so you learn (and get good grades)

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Homework problems are an excellent opportunity to learn mathematics. Here's how to get the most out of them so that you come out having *learned*, and also, with a *good grade*. This necessarily takes more time, but it's time in the bank that will support you later on your exams and everywhere.

Tools and Visualizations

Any Multiplication Table Mod n

Modular Arithmetic Dynamics Grapher

Modular Arithmetic Order Computer

Modular Arithmetic: Sage Guide. Part I

Caesar Cipherer

I give this advice after many years of long experience learning mathematics via homework and independent study. My job description, as a research mathematician and teacher, is at least 50% *learning mathematics*, and also includes teaching students how to learn mathematics.

Here's the method:

- 1. The first rule is to have a rough-draft sheet for doing homework, and a final version sheet. Put the final version sheet away (you won't need it right now). Get out the roughdraft sheet.
- 2. First, read the problem carefully. Figure out what general class of problem it belongs to and what general topics it is drawing on (for example, linear independence, proofs about injectivity, etc.). Locate the resources you have been provided for that (for example, the appropriate textbook chapter).
- 3. Take a few minutes to review the general material. For example, what is the definition of linear independence or injectivity? You don't need to study it in depth, but give yourself time to review the general ideas. This sneaks a little studying into the cracks (so you don't need to cram later) and it prepares you for the problem, so you won't waste time spinning your wheels. It pays off in the end.
- 4. Now read the problem carefully to determine what it is asking, exactly. Figure out what tools you may need, and keep your textbook chapter handy. Look at similar examples in the text and understand them. But importantly: don't write a final solution. Just jot down your ideas in a way that are meaningful to you on your rough-draft sheet. Focus on meaning, not writing. If you get stuck, read more, ask for help, do similar problems, consult the internet for general resources (never google your exact problem; that's cheating after all). Write down your confusions and take them to office hour or ask your teacher after class.
- 5. When you've got the problem solved, read it over and explain it out loud to yourself.

Lessons / Study Guides

Introduction to LaTeX

Modular Arithmetic: User's Manual

Modular Arithmetic: Under the Hood

Modular Arithmetic: In Motion

Modular Arithmetic: Addition in Motion

Blog Posts

LaTeX it!

How to do your homework so you learn (and get good grades)

How to Study Remotely

Reading Actively: Part I

Math Anxiety

Who is that professor at the board, and what does she want?

- 6. Then take our your final copy sheet. Now, importantly, *put* away all your resources, including your rough draft! Then write a beautiful final copy of your solution. Along the way, you are likely to find small errors and correct them, or notice details you didn't take care with, and you can now take care. Do this entirely from your brain!
- 7. If you can't get it all, put away the final draft sheet and go back in the cycle to the studying steps above. When you are confident again, put away your aids and resources and take out the final draft sheet and get it right this time. Repeat as needed.

That's it! But why do I suggest these things? There are principles behind the method. Here they are.

Your brain is fundamentally lazy.

Unless you impose a structure that forces your brain to do math the hard way (with its own neurons) it will rely on patternmatching, short-term memory and other crutches. They may get you through the homework, but won't result in much understanding. It's easier to follow along an example in the text, and you will always find yourself doing this (and eating all the cookies in the cupboard too) unless you set yourself concrete rules. Think of your brain as a beloved but lazy pet dog.

You only learn things that go via your brain.

It is really easy to work step-by-step through a similar problem, understanding each step and doing the analogous thing on your homework, without truly internalizing. I know this from long personal experience. When preparing a lecture, if I copy the textbook proof into my notes sentence-by-sentence, I'll often end up at the board confused about why some step was the way it was. However, if I get the main idea, close the text, and produce my own proof, then I will own the material and won't get confused. Even better if I have the time to invent the proof from scratch by myself!

To make sure understanding goes via the brain, the key tricks are

Home	More Info	
	Home	

About

time and space.

Time: work out the complete solution with aids, then put the aids away and work through the entire thing in final draft without them. If the ideas stay in your brain (instead of on paper) for the length of time this process takes, then they are actually stored in your brain as generative understanding, not just in short-term memory of some variable names and equations.

Space: without any aids as you work (they are put away in physical space) you are required to dig up the answer using only your brain, and this is exactly the moment you are learning the material (strange as it may seem!). You may remember a hint of how the argument went, but without aids, you are forced to figure out how the argument went instead of simply remembering it in short-term memory for the ten seconds while you turn from the textbook to your notebook.

Figuring things out is when you learn.

You don't learn while you are reading or listening. That's just taking in information for processing. But the processing is where the magic is. You learn when you disassemble, analyze, organize and justify. So the whole point of this method is to force that activity as much as possible. And it doesn't happen anywhere more than when you are trying to reproduce, from your own understanding, a mathematical argument on a blank page with no aids.

Short term memory, rote memorization, and pattern matching are not always your friends.

These are useful aspects of the human brain, but they often do not represent true understanding. True understanding means constructing the *why* of something for yourself. Never memorize, and don't rely on pattern matching as an explanation for anything.

And finally...

And finally, you'll get better grades too, because your homework will be *more correct*, and *more legible*! (Don't discount the grade impact of neat organized homework.) And you'll be more prepared for your exams, before you even begin studying for them, because you will have learned more!

Read more about forcing your brain to be active here.

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