MIDTERM
LINEAR ALGEBRA

MATH 2130
SUMMER 2018

Friday June 15, 2018
9:15 AM – 10:50 AM

Name

PRACTICE EXAM

Please answer all of the questions, and show your work.
You must explain your answers to get credit.
You will be graded on the clarity of your exposition!

1	2	3	4	5	6	7	8	
10	10	10	10	10	10	10	10	total

Date: June 14, 2018.

1. *Find all solutions to the following system of linear equations:*

1
10 points

$$\begin{array}{rcrcrcr} 3x_1 & + & 9x_2 & + & 27x_3 & = & -3 \\ -3x_1 & - & 11x_2 & - & 35x_3 & = & 5 \\ 2x_1 & + & 8x_2 & + & 26x_3 & = & -4 \end{array}$$

2. Consider the matrix

2
10 points

$$A = \begin{bmatrix} 1 & -3 & 0 & -1 & 4 & -2 \\ 0 & 0 & 1 & -1 & 0 & 1 \\ 3 & -9 & 0 & -3 & 2 & 4 \\ 1 & -3 & 1 & -2 & 4 & -1 \end{bmatrix}$$

2.(a). *Find the reduced row echelon form of A.*

2.(b). *Are the columns of A linearly independent?*

2.(c). *Are the rows of A linearly independent?*

2.(d). *What is the column rank of A?*

2.(e). *What is the row rank of A?*

3. Consider the linear map $L : \mathbb{R}^3 \to \mathbb{R}^2$ given by

3
10 points

$$L(x_1, x_2, x_3) = (2x_1 - x_3, 3x_2 + x_3).$$

Write down the matrix form of the linear map L.

4
10 points

4. Consider the matrix

$$B = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

4.(a). *Find the inverse of B.*

4.(b). *Does there exist $x \in \mathbb{R}^3$ such that $Bx = \begin{bmatrix} 5 \\ \sqrt{2} \\ \pi \end{bmatrix}$?*

5. Suppose that

5
10 points

$$C = \begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ -4 & -5 & 3 & -8 & 1 \\ 2 & -5 & -4 & 1 & 8 \\ -6 & 0 & 7 & -3 & 1 \end{bmatrix}$$

Find a lower triangular matrix L and an echelon form matrix U such that $C = LU$.

6. **TRUE** or **FALSE**: *Suppose that* $V \subseteq \mathbb{R}^n$ *is a subset satisfying:*

6
10 points

(1) *For all* $v_1, v_2 \in V$, *we have* $v_1 + v_2 \in V$.
(2) *For all* $v \in V$, *we have* $-v \in V$.

Then V *is a subspace of* $\mathbb{R}^n$.

7
10 points

7. The equation $\mathbf{x} = C\mathbf{x} + \mathbf{d}$ (the Leontief Production Equation) arises in the Leontief Input-Output Model. Here $\mathbf{x}, \mathbf{d} \in M_{n\times 1}(\mathbb{R})$ are column vectors and $C \in M_{n\times n}(\mathbb{R})$ is a square matrix. Consider also the equation $\mathbf{p} = C^T\mathbf{p} + \mathbf{v}$ (called the price equation), where $\mathbf{p}, \mathbf{v} \in M_{n\times 1}(\mathbb{R})$ are column vectors.

Show that

$$\mathbf{p}^T\mathbf{d} = \mathbf{v}^T\mathbf{x}.$$

(This quantity is known as GDP.) [Hint: Compute $\mathbf{p}^T\mathbf{x}$ in two ways.]

8
10 points

8. **TRUE** or **FALSE**. You do **not** need to justify your answer.

8.(a). Let $A \in M_{m \times n}(\mathbb{R})$. There is an $x \in \mathbb{R}^n$ such that $Ax = 0$.

T F

8.(b). Let $A \in M_{m \times n}(\mathbb{R})$. If the columns of A span $\mathbb{R}^m$, then for any $b \in \mathbb{R}^m$ there is an $x \in \mathbb{R}^n$ such that $Ax = b$.

T F

8.(c). The map $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$ for all $x \in \mathbb{R}$ is a linear map.

T F

8.(d). If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then $A^n = \begin{bmatrix} 1 & 2^{n-1} \\ 0 & 1 \end{bmatrix}$ for each natural number n.

T F

8.(e). If A and B are $m \times n$ matrices, then $A + B = B + A$.

T F

8.(f). Let $A \in M_{m \times n}(\mathbb{R})$. If the rows of A are linearly independent, then for any $b \in \mathbb{R}^m$ there is at most one $x \in \mathbb{R}^n$ such that $Ax = b$.

T F

8.(g). Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be a linear map. The kernel of f is a sub-vector space of $\mathbb{R}^n$.

T F

8.(h). If the columns of a square matrix A are linearly independent, then A^T is invertible.

T F

8.(i). If A is a square matrix such that sums of the absolute values of the entries of each column of A is less than 1, then $(\mathrm{Id} - A)$ is invertible.

T F

8.(j). Suppose that A and B are square matrices, and AB is invertible. Then A and B are invertible.

T F