

§6.6 Part I: Work

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Key Points:

- $W = \text{Force} \times \text{Distance} = F \cdot d$

- Units:

| | $F = \text{Force}$ | $d = \text{Distance}$ | $W = \text{Work}$ |
|------------|--------------------|-----------------------|-------------------|
| Metric | | | |
| U.S. Units | | | |

- Now, what if F is not constant?

- Dealing with springs - **Hooke's Law:**

$$F = kx,$$

where x is the distance stretched or compressed past the natural (equilibrium) length, and k is the spring constant.

- Dealing with the force of gravity (metric system):

$$F = mg,$$

where m is the mass of the object and $g = 9.8 \frac{\text{m}}{\text{sec}^2}$.

- Dealing with the force of gravity (U.S. system):

$$F = \frac{W}{g}.$$

Examples:

1. A box is slid 3 meters across a carpet against a force of kinetic friction of 45N. How much work is done?

2. I am pushing my sister across a 10 foot room. She pushes back with increasing ferocity, with a force of $20 + \frac{x^2}{2}$ pounds, where x is how far I have pushed her. How much work do I do?

3. A 30-centimeter long spring with a spring constant of $k = 120 \frac{\text{N}}{\text{m}}$ is compressed to 20cm. Calculate the work done.

4. A force of 10 lbs is required to hold a spring stretched to 6 inches past its natural length. Calculate the work required to stretch it 8 inches past its natural length.

5. How much energy is required to hoist a 3-kilogram pumpkin 15 meters to the roof of the math building?

6. How much energy is required to carry a 44-lb stack of books up to the third floor of the math building? (30 ft.)

7. A 6-kg chain is 3 meters long. How much work is done lifting it from the ground until its lower end is 2 meters off of the ground?

8. How much work is done emptying a $2 \times 2 \times 3$ -ft rectangular tank? The water must be pumped to a point in the upper corner of the tank.

9. A tub has the shape of the solid of revolution formed by rotating around the y -axis the portion of the curve $y = 2x^4$ that lies between $x = 0$ and $x = 1$. (Draw a picture.) How much work is done to empty the tank? All of the water must be pumped out of the top of the tank.