Intro to Parametric Equations (see §1.7)

With inspiration from Faan Tone

Key Points:

- The coordinates x and y are each defined separately as functions of time t. Together, they trace out a curve in the plane. The two equations for x and y are called **parametric equations** and t is called the **parameter**.
- Think of a bug (or a snail!) crawling on the plane. $x = x(t)$ and $y = y(t)$ keep track of its x- and y-position over time.
- Sometimes, we can eliminate the parameter to write an equation involving only x and y.

  Alert! This can help us to draw a graph, but loses all information about movement along the graph. (e.g. We can see the slime trail left by the snail, but we don't know how fast it went or in which direction.)

Examples:

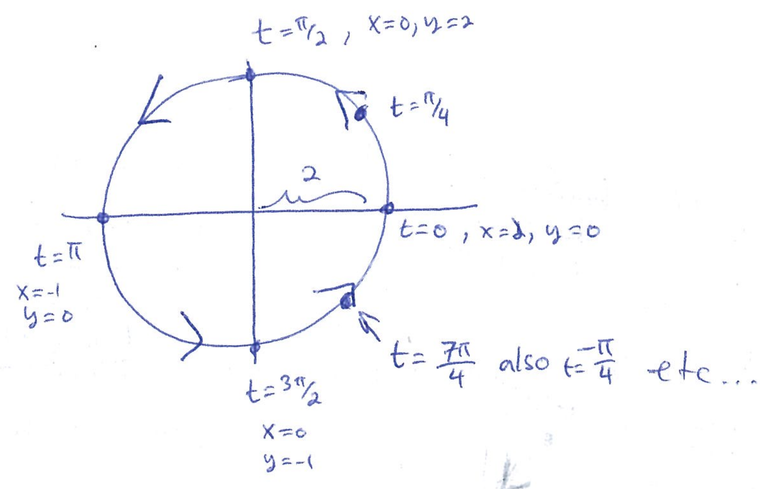
1. (a) Sketch the curve defined by the parametric equations

$$\begin{cases} x(t) = 2\cos(t) \\ y(t) = 2\sin(t). \end{cases}$$

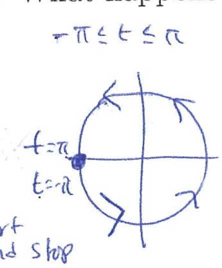
Indicate with an arrow the direction in which the curve is traced as t increases.

t	x	y
0	2	0
$\pi/4$	$\sqrt{2}$	$\sqrt{2}$
$\pi/2$	0	2
π	-2	0
$-\pi/4$	$\sqrt{2}$	$-\sqrt{2}$

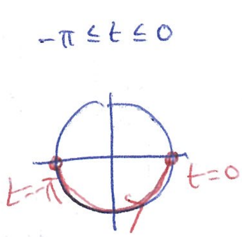
Can also do negative time



(b) What happens to our picture if we make the restriction $-\pi \le t \le \pi$? ⇐ I wanted this to be something like $-\pi \le t \le 0$



Start and stop at pt $(x,y) = (-2,0)$



2. (a) Sketch the curve defined by the parametric equations

$$\begin{cases} x(t) = t^2 - 2t \\ y(t) = t + 1. \end{cases}$$

Could also plug in pts. But this other way gives more info.

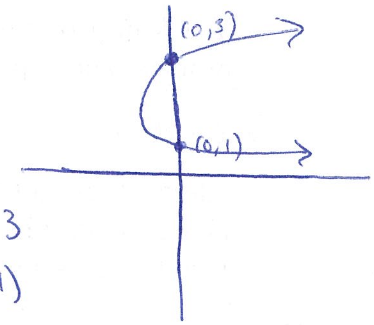
One way is to Solve for t:

$y - 1 = t$

$X = (y-1)^2 - 2(y-1)$
$= y^2 - 2y + 1 - 2y + 2$
$= y^2 - 4y + 3$

$\Rightarrow$ $X = y^2 - 4y + 3$
$X = (y-3)(y-1)$

sideways (right-opening) parabola.



(b) Eliminate the parameter t to find an equation for the graph of this curve in terms of x and y only.

Oops. Already did this in part (a)

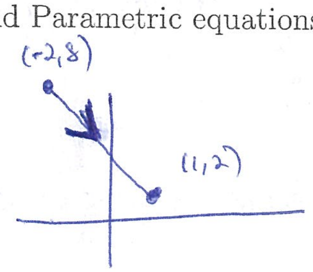
Another way is to do a mixture of $(-2,8)$ and $(1,2)$:

$(-2,8)\cdot(1-t) + t\,(1,2)$ for $0 \le t \le 1$
$= (-2+2t,\ 8-8t) + (t, 2t)$
$= (-2+3t,\ 8-6t)$

$\boxed{\begin{cases} x(t) = -2+3t \\ y(t) = 8-6t \end{cases} \quad 0 \le t \le 1}$

Preview of Next Time!

3. Find Parametric equations for a line segment from $(-2, 8)$ to $(1, 2)$.



A third way is to use calculus:

~~x(t)~~

$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{-2}{1}$ ← slope

$\begin{cases} y'(t) = -2 \\ x'(t) = 1 \end{cases}$

$\begin{cases} y(t) = -2t + a \\ x(t) = t + b \end{cases}$

then solve for a, b and the time interval like in Method I.

Motion is linear in X and in y.

$\begin{cases} x(t) = m_1 t + a \\ y(t) = m_2 t + b \end{cases}$

Slope is $\frac{\Delta y}{\Delta x} = \frac{8-2}{-2-1} = -2$

Could write $-2 = \frac{-2}{1} \frac{\Delta y}{\Delta x}$

$\begin{cases} x(t) = 1t + a \\ y(t) = -2t + b \end{cases}$

use that $X(0) = (-2, 8)$ to find $a = -2$, $b = 8$

$\begin{cases} x(t) = t - 2 \\ y(t) = -2t + 8 \end{cases}$

Find ends t:
$2 = -2t + 8 \rightarrow t = 3$ ✓
$1 = t - 2 \rightarrow t = 3$ ✓

$\boxed{\begin{cases} X(t) = t - 2 \\ y(t) = -2t + 8 \end{cases} \quad 0 \le t \le 3}$

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