Intro to Parametric Equations (see §1.7)

With inspiration from Faan Tone

Key Points:

- The coordinates x and y are each defined separately as functions of time t. Together, they trace out a curve in the plane. The two equations for x and y are called **parametric equations** and t is called the **parameter**.
- Think of a bug (or a snail!) crawling on the plane. $x = x(t)$ and $y = y(t)$ keep track of its x- and y-position over time.
- Sometimes, we can eliminate the parameter to write an equation involving only x and y.

  Alert! This can help us to draw a graph, but loses all information about movement along the graph. (e.g. We can see the slime trail left by the snail, but we don't know how fast it went or in which direction.)

Examples:

1. (a) Sketch the curve defined by the parametric equations

$$\begin{cases} x(t) = 2\cos(t) \\ y(t) = 2\sin(t). \end{cases}$$

   Indicate with an arrow the direction in which the curve is traced as t increases.

   (b) What happens to our picture if we make the restriction $-\pi \leq t \leq \pi$?

2. (a) Sketch the curve defined by the parametric equations

$$\begin{cases} x(t) = t^2 - 2t \\ y(t) = t + 1. \end{cases}$$

(b) Eliminate the parameter t to find an equation for the graph of this curve in terms of x and y only.

3. Find Parametric equations for a line segment from $(-2, 8)$ to $(1, 2)$.