§8.7: Taylor's Inequality

(Created by Faan Tone Liu)

Key Points:

- Goal: Find the size of the error when using a Taylor Polynomial to estimate a function. The error (remainder) is $R_n(x) = f(x) T_n(x)$, where $T_n(x)$ is the *n*th degree Taylor polynomial of f.
- The size of the error is influenced by the following:
 - the degree n of the Taylor Polynomial
 - the distance between x and the point a around which the Taylor Series is centered
 - the size of $|f^{(n+1)}(x)|$ in the interval between x and a.
- Taylor's Inequality: If $f^{(n+1)}$ is continuous and $|f^{(n+1)}(x)| \leq M$ between x and a, then the remainder $R_n(x)$ satisfies the inequality

$$|R_n(x)| \le \frac{M}{(n+1)!} |x-a|^{n+1}$$

• Procedure: Find M and fill in a. Then $R_n(x)$, n and x interact. You'll typically be given two of them and have to find the third.

Examples:

1. Use the 6th degree Tayolor Polynomial for e^x , centered at a=0 to estimate e. How accurate is your estimate guaranteed to be?

2. How accurate is the 3rd degree Taylor Polynomial centered at 0 guaranteed to be in estimating $\sin(9^{\circ})$.

3. Show that the Taylor Series for cos(x) centered at a = 0 converges to cos(x).

4. How large should n be to estimate e to within six decimal places (0.0000005) using a Taylor Polynomial centered at x = 0.

5. Use a 5th degree Taylor Polynomial to estimate ln(1.1). How accurate is your estimate?