§8.4: Absolute Convergence and Ratio Test

(Thanks to Faan Tone Liu)

Key Points:

- If the series $\sum |a_n|$ converges, then the original series $\sum a_n$ converges:

  In this case, we say the series $\sum a_n$ is **absolutely convergent** or **converges absolutely**. This is a new way to prove series convergence!

- Series that are convergent but are not absolutely convergent are called **conditionally convergent**. One example is

- The **Ratio Test** is another tool we can use. Calculate $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L$.

  (i) If $L < 1$, then $\sum a_n$ converges absolutely (and thus converges)

  (ii) If $L > 1$, including $+\infty$, then $\sum a_n$ diverges

  (iii) If $L = 1$, the ratio test tells you nothing. Try something else.

- Helpful graphic:

	$\sum a_n$ converges	$\sum a_n$ diverges
$\sum \|a_n\|$ converges		
$\sum \|a_n\|$ diverges		

Examples:

1. Does $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$ converge or diverge?

2. Does $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converge or diverge?

3. Does $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ converge absolutely, converge conditionally, or diverge?

4. Does $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ converge absolutely, converge conditionally, or diverge?

5. Does $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n!}{e^n}$ converge absolutely, converge conditionally, or diverge?

6. Does $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$ converge absolutely, converge conditionally, or diverge?