Daily Quiz

- Go to Socrative.com and complete the quiz.
- Room Name: HONG5824
- Use your full name.

5.7 Partial Fractions

Find $\displaystyle\int \frac{5x-4}{2x^2+x-1}\,dx.$

* The goal of partial fractions is to break up a rational function into sums of simpler fractions that are easier to integrate.

Observe that the denominator can be factored:

$$\frac{5X-4}{2X^2+X-1} = \frac{5X-4}{(X+1)(2X-1)}$$

Since the numerator has a smaller degree than the denominator, we will be able to decompose the rational function as shown below:

$$\frac{5X-4}{(X+1)(2X-1)} = \frac{A}{X+1} + \frac{B}{2X-1}$$ where A and B are constants.

we can algebraically solve for A and B as follows:

① Clear the denominator

$$\frac{5x-4}{(x+1)(2x-1)}(x+1)(2x-1) = \frac{A}{x+1}(x+1)(2x-1) + \frac{B}{2x-1}(x+1)(2x-1)$$

$$5x-4 = A(2x-1) + B(x+1)$$

② Collect all the coefficients of different powers of x (note that constants count as having $x^0=1$ as a power of x)

$$5x-4 = 2Ax - A + Bx + B$$

$$5x-4 = x[2A+B] + [-A+B]$$

③ set the coefficients on the left-hand side equal to those on the right-hand side.

$$5 = 2A+B$$
$$-4 = -A+B$$

④ Solve for A and B using the equations from step ③.

$5 = 2A + B$
$-4 = -A + B$

$9 = 3A$
$3 = A$

$-4 = -3 + B$
$-1 = B$

⑤ Integrate

$$\int \frac{5x-4}{(x+1)(2x-1)}dx = \int \frac{3}{x+1}dx + \int \frac{-1}{2x-1}dx$$

simple u-sub   simple u-sub

$$= \boxed{3\ln|x+1| - \frac{1}{2}\ln|2x-1| + C}$$

Alternate Method of obtaining A and B (Heaviside / Residue Method)
(only works for degree 1 factors)

Let $f(x) = \dfrac{5x-4}{(x+1)(2x-1)}$.
$$\frac{5x-4}{(x+1)(2x-1)} = \frac{A}{x+1} + \frac{B}{2x-1}$$

$$A = \lim_{x\to -1} (x+1) f(x) = \lim_{x\to -1} \frac{5x-4}{2x-1} = \frac{-5-4}{-2-1} = \frac{-9}{-3} = 3$$

Here -1 is the root of $(x+1)$.

$$B = \lim_{x\to \frac{1}{2}} (2x-1) f(x) = \lim_{x\to \frac{1}{2}} \frac{5x-4}{x+1} = \frac{5/2-4}{\frac{1}{2}+1} = \frac{-3/2}{3/2} = -1$$

Here $\frac{1}{2}$ is the root of $(2x-1)$.

5.7 Partial Fractions

Find $\displaystyle\int \frac{x^2+2x-1}{x^3-x}\,dx.$ Let $f(x)=\dfrac{x^2+2x-1}{x^3-x}$

Factor the denominator:

$$x^3-x = x(x^2-1) = x(x-1)(x+1)$$

$$\frac{x^2+2x-1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

Compute the residues

$$A = \lim_{x\to 0} x\,f(x) = \lim_{x\to 0} \frac{x^2+2x-1}{(x-1)(x+1)} = \frac{0+0-1}{-1\cdot 1} = 1$$

$$B = \lim_{x\to 1} (x-1)f(x) = \lim_{x\to 1} \frac{x^2+2x-1}{x(x+1)} = \frac{1+2-1}{1\cdot 2} = 1$$

$$C = \lim_{x\to -1} (x+1)f(x) = \lim_{x\to -1} \frac{x^2+2x-1}{x(x-1)} = \frac{1-2-1}{-1(-2)} = -1$$

5.7 Partial Fractions

Find $\displaystyle\int \frac{x^2+2x-1}{x^3-x}\,dx.$

Hence $\dfrac{x^2+2x-1}{x(x-1)(x+1)} = \dfrac{1}{x} + \dfrac{1}{x-1} + \dfrac{-1}{x+1}$

Integrate:

$$\int \frac{x^2+2x-1}{x^3-x}\,dx = \int \frac{1}{x}\,dx + \int \frac{1}{x-1}\,dx + \int \frac{-1}{x+1}\,dx$$

simple u-substitution

$$= \boxed{\ln|x| + \ln|x-1| - \ln|x+1| + C}$$