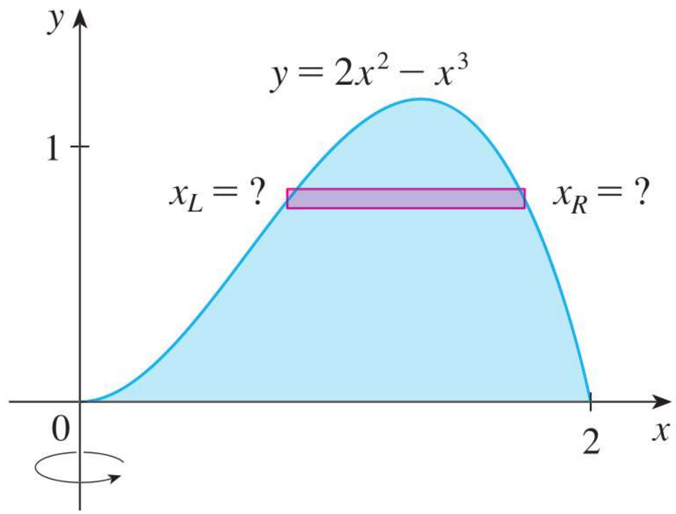
Daily Quiz

- Go to Socrative.com and complete the quiz.
- Room Name: HONG5824
- Use your full name.

6.3 Volumes by Cylindrical Shells



- Using cross-sections requires us to solve for x or y.
- What if we can't solve for the other variable?
- Can we solve for x with the given function below?

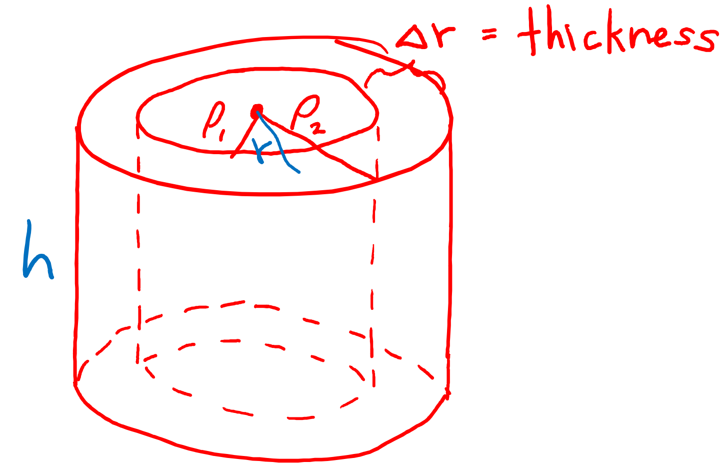
$$y = 2x^2 - x^3$$

6.3 Cylindrical Shells

ρ_2 = big radius $\Delta r = \rho_2 - \rho_1$

ρ_1 = small radius

$r = \dfrac{\rho_1 + \rho_2}{2}$ = average radius



volume of the hollow shell

$$= \pi \rho_2^2 h - \pi \rho_1^2 h$$

$$= \pi h (\rho_2^2 - \rho_1^2)$$

$$= \pi h (\rho_2 + \rho_1)(\rho_2 - \rho_1)$$

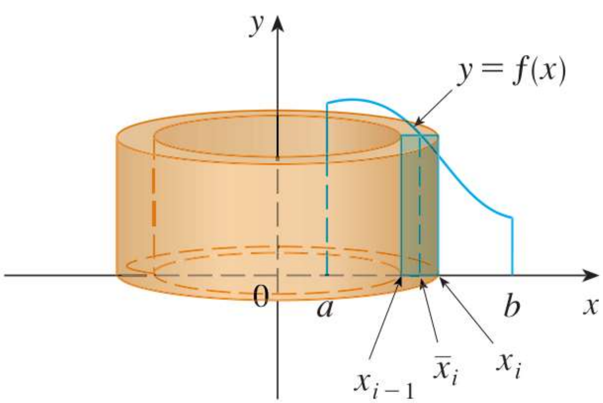
$$= 2\pi h \left(\frac{\rho_2 + \rho_1}{2}\right)(\rho_2 - \rho_1)$$

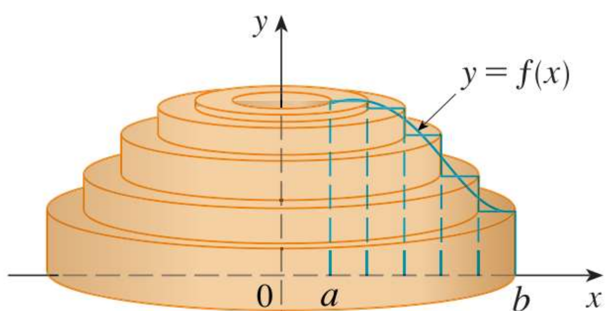
$$= \underbrace{2\pi r h}_{\text{surface area of the shell}} \ \underbrace{\Delta r}_{\text{thickness}}$$

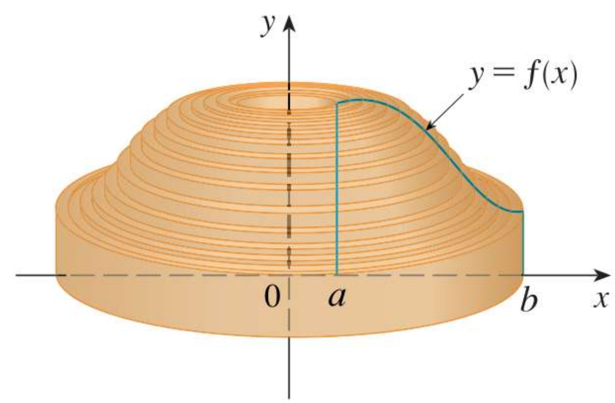
6.3 Volumes by Cylindrical Shells

- Instead of cutting slices, let's peel one layer at a time like with onions.

$$V \approx \underbrace{\sum_{i=1}^{n}}_{\text{sum}} \overbrace{SA(x_i)}^{\text{surface area of the shell}} \underbrace{\triangle x}_{\text{thickness}} \qquad V = \underbrace{\int_a^b}_{\text{integral}} \overbrace{SA_x}^{\text{surface area of the shell}} \underbrace{dx}_{\text{thickness}}$$



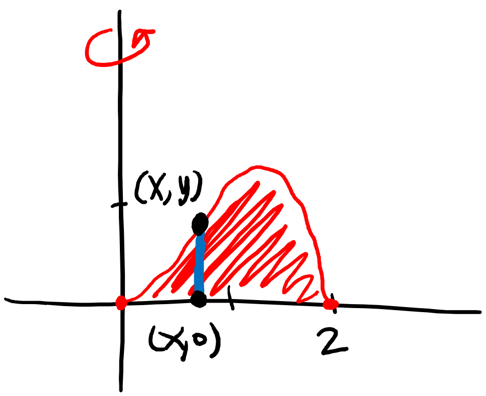


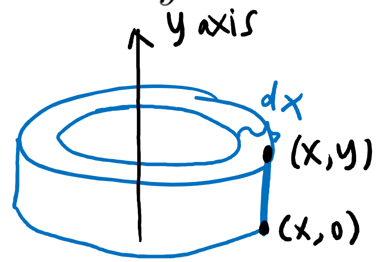


6.3 Volume by Cylindrical Shells

Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = 2x^2 - x^3$ and $y = 0$.

$y = x^2(2-x)$





height = y

radius = x

surface area = $2\pi xy$

volume of shell = $2\pi xy\,dx$

Total volume = $\int_0^2 2\pi xy\,dx$

$$= 2\pi \int_0^2 xy\,dx = 2\pi \int_0^2 x(2x^2 - x^3)\,dx = 2\pi \int_0^2 2x^3 - x^4\,dx$$

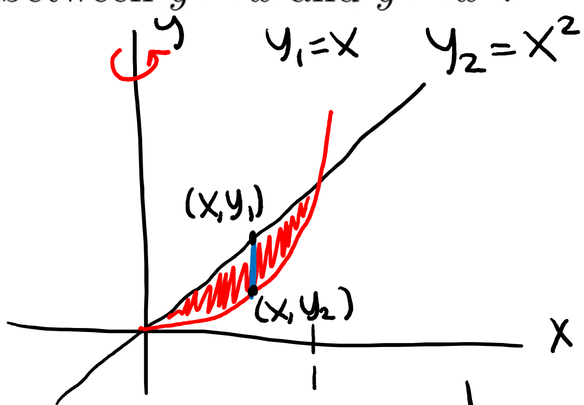
$$= 2\pi \left[\frac{2x^4}{4} - \frac{x^5}{5}\right]_0^2$$

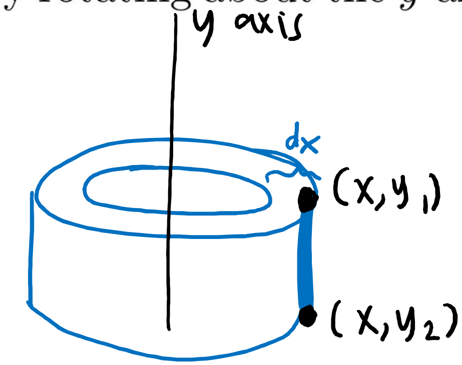
$$= 2\pi \left[\frac{16}{2} - \frac{32}{5}\right)$$

$$= \frac{16\pi}{5}$$

6.3 Volume by Cylindrical Shells

Find the volume of the solid obtained by rotating about the y-axis the region between $y = x$ and $y = x^2$.





height $= y_1 - y_2$

radius $= x$

Surface area $= 2\pi x (y_1 - y_2)$

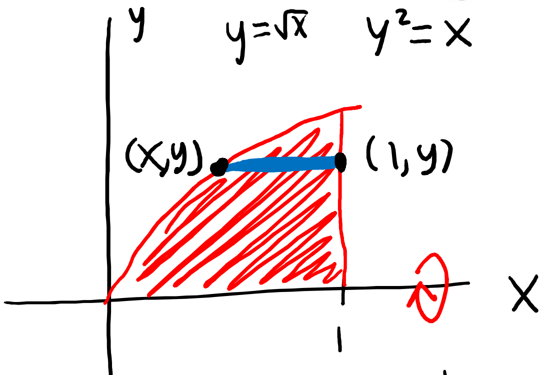
volume of shell $= 2\pi x (y_1 - y_2)\, dx$

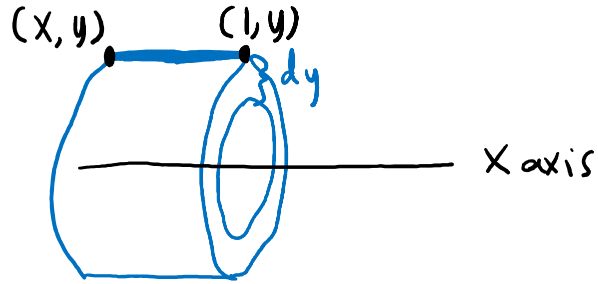
$$\text{Total volume} = \int_0^1 2\pi x (y_1 - y_2)\, dx = 2\pi \int_0^1 x(x - x^2)\, dx = 2\pi \int_0^1 x^2 - x^3\, dx$$

$$= 2\pi \left[\frac{x^3}{3} - \frac{x^4}{4} \right]_0^1 = 2\pi \left[\frac{1}{3} - \frac{1}{4} \right] = \frac{\pi}{6}$$

6.3 Volume by Cylindrical Shells

Use cylindrical shells to find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ from 0 to 1.





height $= 1-x$

radius $= y$

surface area $= 2\pi y(1-x)$

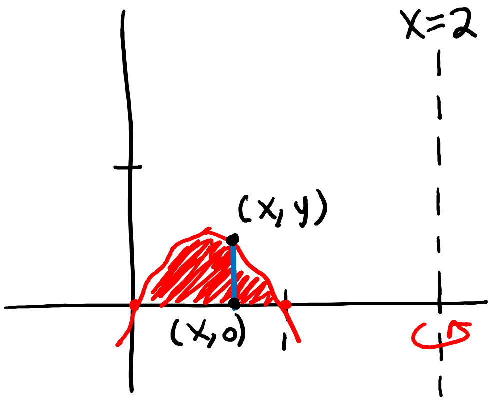
volume of shell $= 2\pi y(1-x)\,dy$

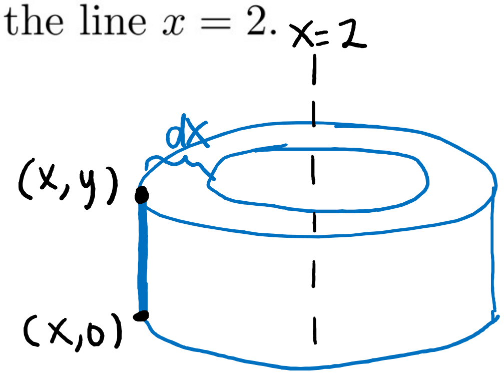
$$\text{Total volume} = \int_0^1 2\pi y(1-x)\,dy = 2\pi\int_0^1 y(1-y^2)\,dy = 2\pi\int_0^1 y-y^3\,dy$$

$$= 2\pi\left[\frac{y^2}{2}-\frac{y^4}{4}\right]_0^1 = 2\pi\left[\frac{1}{2}-\frac{1}{4}\right] = \frac{\pi}{2}$$

6.3 Volume by Cylindrical Shells

Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$.





height = y

radius = $2-x$

surface area = $2\pi(2-x)y$

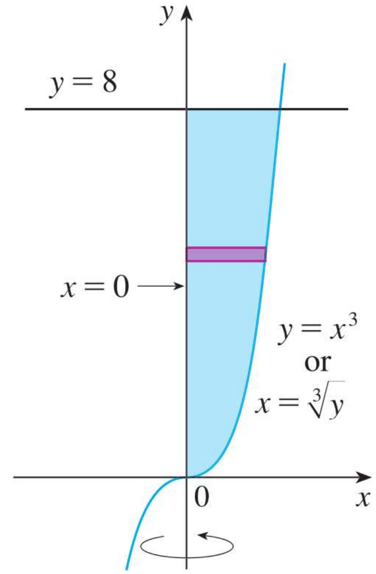
volume of shell = $2\pi(2-x)y\,dx$

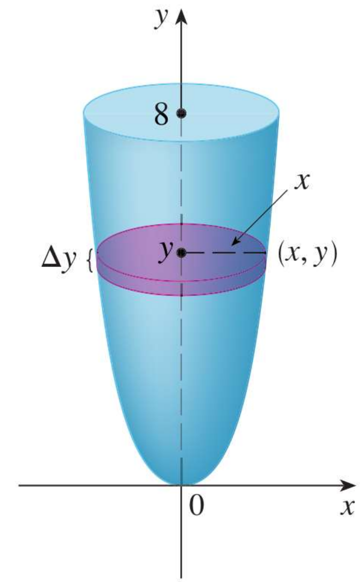
$$\text{Total volume} = \int_0^1 2\pi(2-x)y\,dx = 2\pi\int_0^1 (2-x)(x-x^2)\,dx$$

$$= 2\pi\int_0^1 2x - x^2 - 2x^2 + x^3\,dx = 2\pi\int_0^1 2x - 3x^2 + x^3\,dx = 2\pi\left[x^2 - x^3 + \frac{x^4}{4}\right]_0^1$$

$$= 2\pi\left[1 - 1 + \frac{1}{4}\right] = \frac{\pi}{2}$$

- For the **disk/washer method**, the cuts are **perpendicular** to the axis of rotation. (Example: y-axis is the axis of rotation.)





- For the **shell method**, the cuts are **parallel** to the axis of rotation. (Example: y-axis is the axis of rotation.)