Daily Quiz

- Go to Socrative.com and complete the quiz.
- Room Name: HONG5824
- Use your full name.

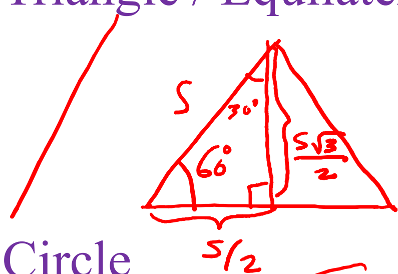
6.2 Shapes

- What are some area formulas?
  - Rectangle

$A = \text{base} \cdot \text{height}$

  - Triangle / Equilateral Triangle

$A = \frac{1}{2} b h$

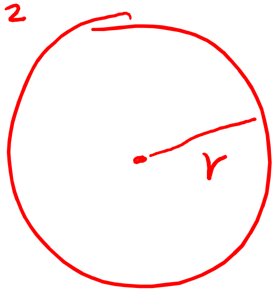


$A = \frac{1}{2}\, s \left(s\frac{\sqrt{3}}{2}\right)$

$= \frac{s^2\sqrt{3}}{4}$

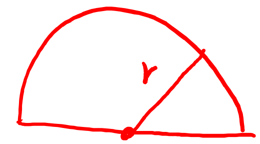
  - Circle

$\pi \cdot r^2$



- Semi-circle

$\frac{1}{2}\pi r^2$

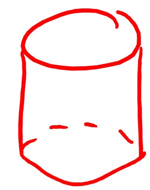


- Surface area of a cylinder

SA of body $= \pi r^2 h$

SA of lids $= 2\pi r^2$

Total $= \pi r^2 + 2\pi r^2$

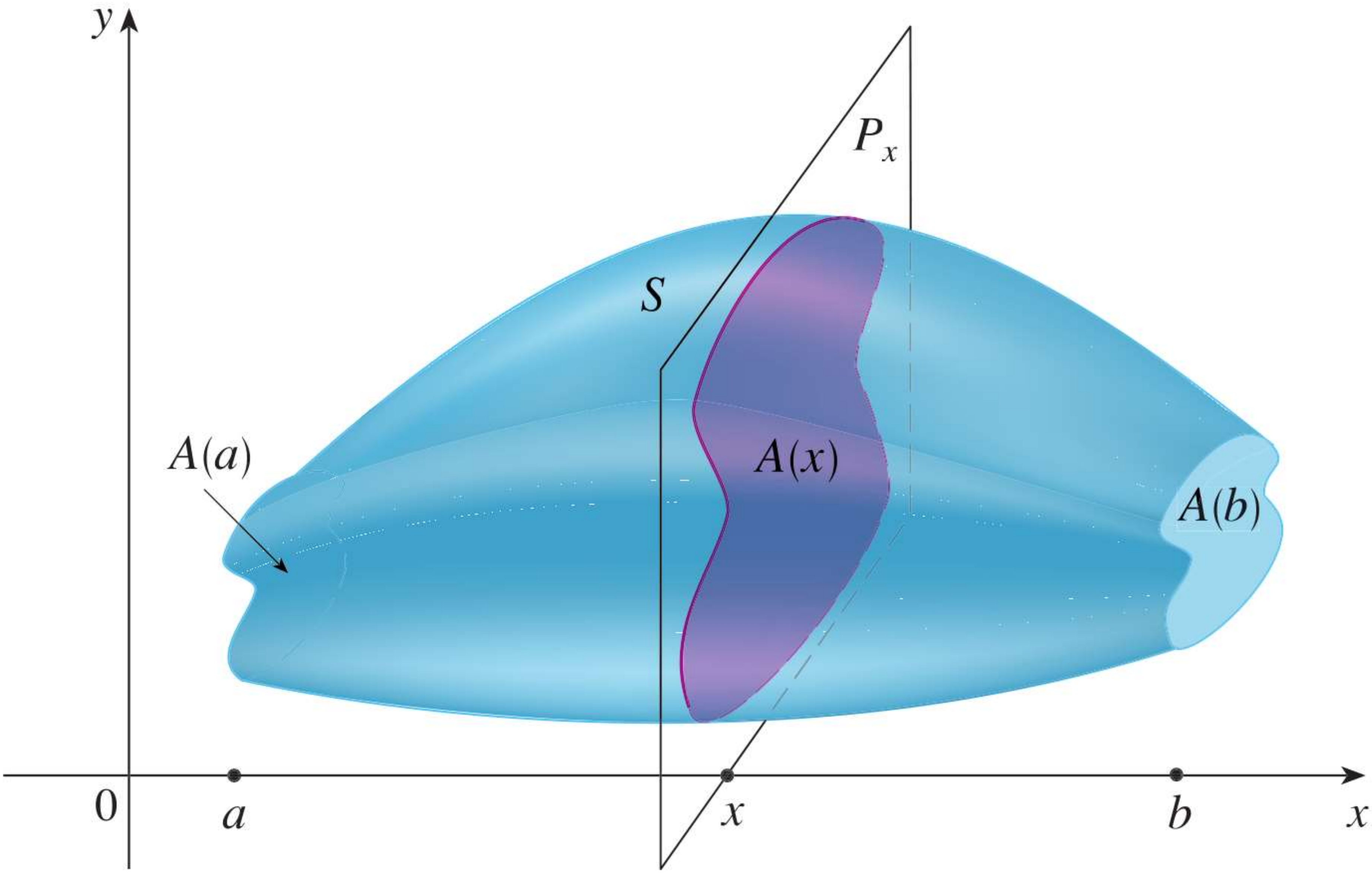


- Equation of a circle centered at the origin of radius r

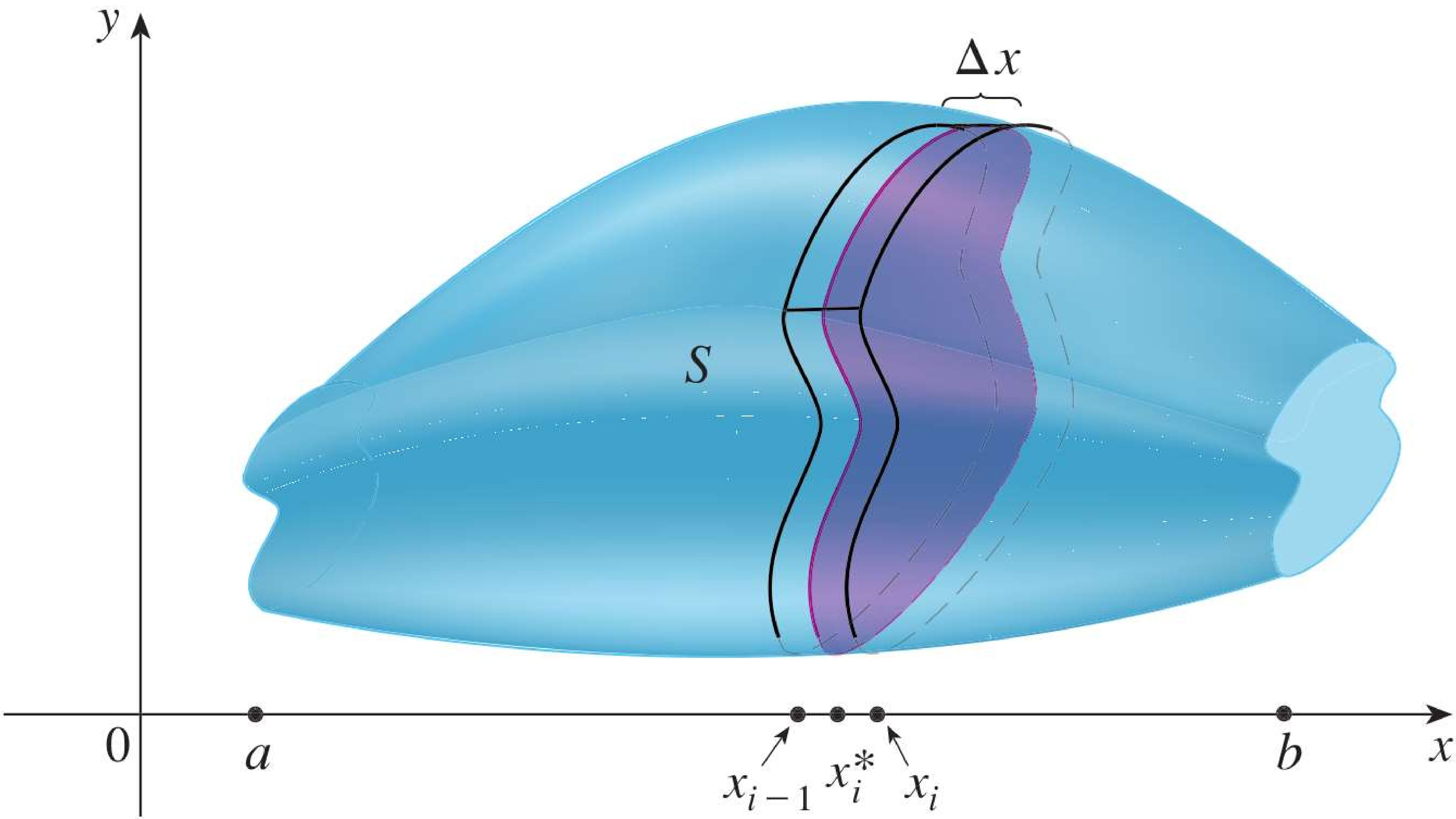
$x^2 + y^2 = r^2$

6.2 Volumes Using Cross-sections

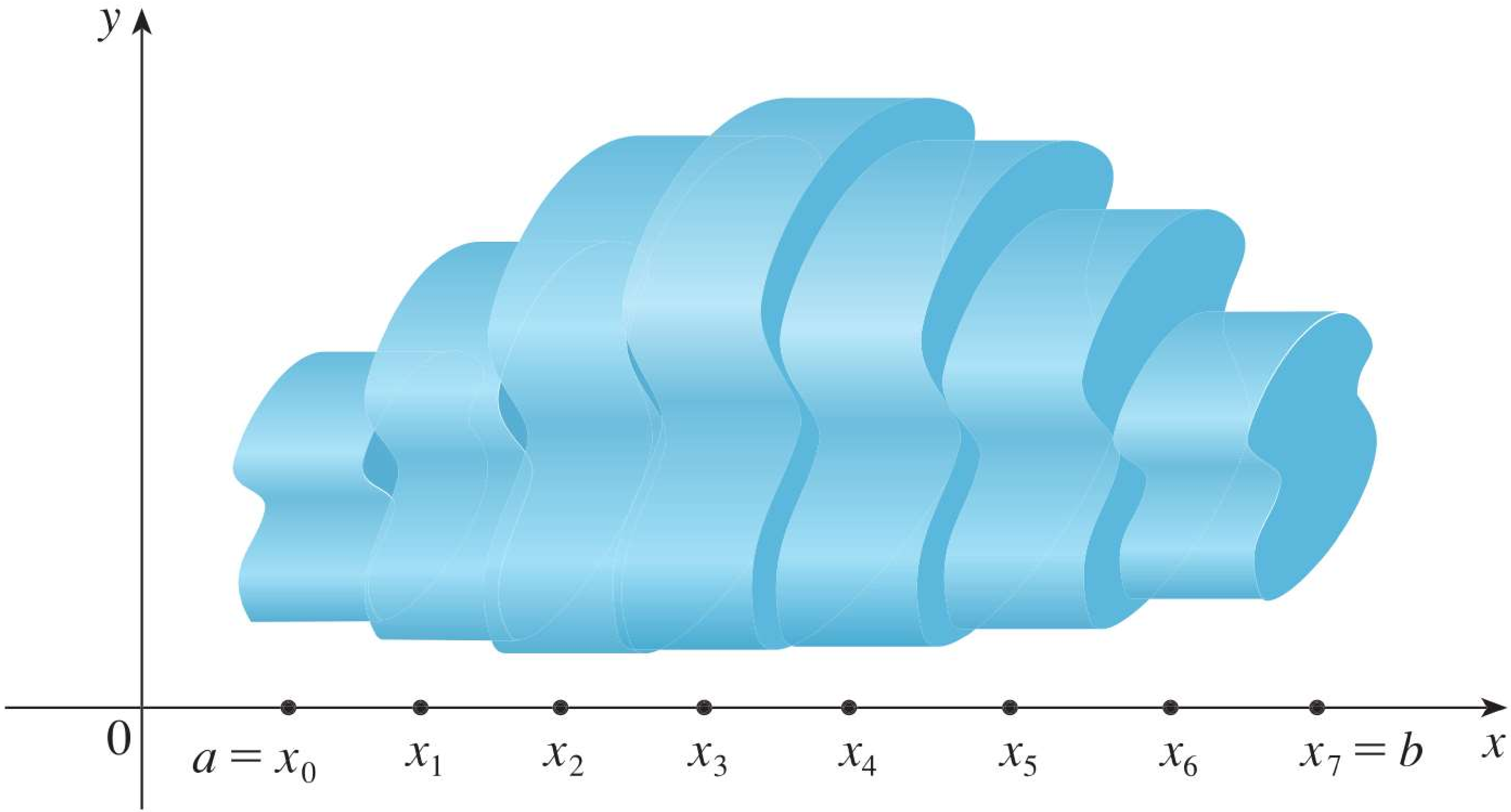
- Suppose we want to compute the volume of a 3-dimensional object.
- We can approximate the volume by using a 3-dimensional analogue of the Riemann sum; we slice the object to find its cross-section and give it a small thickness.
- Adding up the volumes of the slabs, we get an approximation of the object's volume.



y
P_x
S
A(a)
A(x)
A(b)
0
a
x
b
x



y
Δx
S
0
a
x_{i-1}
x_i^*
x_i
b
x


y
0
$a = x_0$
x_1
x_2
x_3
x_4
x_5
x_6
$x_7 = b$
x

6.2 Volumes using Cross-Sections

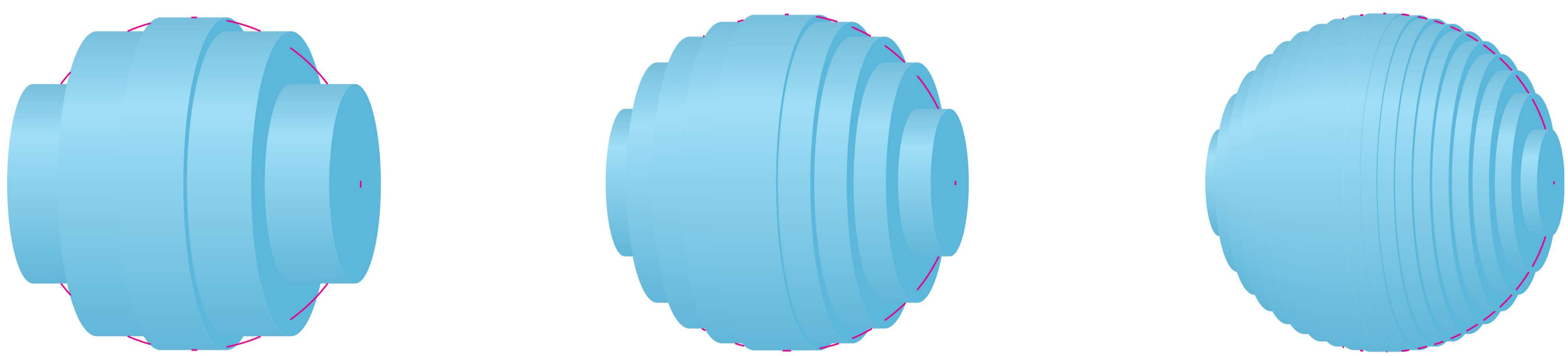
$$V \approx \underbrace{\sum_{i=1}^{n}}_{\text{sum}} \overbrace{A(x_i)}^{\text{Area of the cross-section at } x_i} \underbrace{\triangle x}_{\text{thickness}}$$

$$V = \underbrace{\int_a^b}_{\text{integral}} \overbrace{A_x}^{\text{area of the cross-section at } x} \underbrace{dx}_{\text{thickness}}$$

6.2 Finding Volumes using Cross-Sections

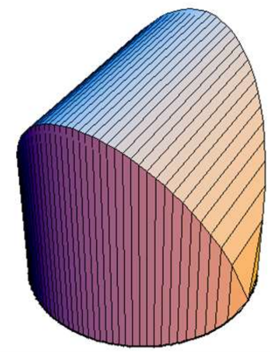
- We can find the exact volume of an object if we take very thin slices and add them up (Sum turns into an integral)

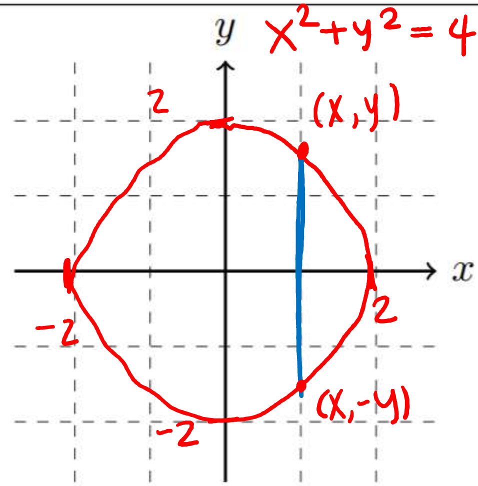
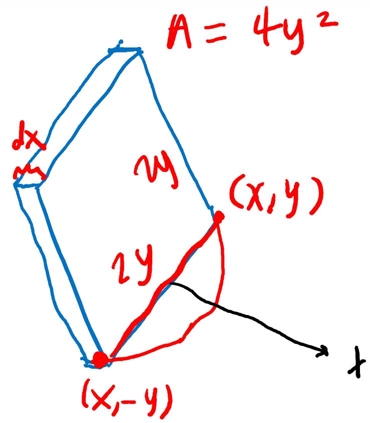
$$V = \int_a^b \underbrace{A_x\, dx}_{\text{volume of a thin slice}} \text{ (integrate along the x-axis)} \qquad V = \int_c^d A_y\, dy \text{ (integrate along the y-axis)}$$



(a) Using 5 disks, $V \approx 4.2726$ (b) Using 10 disks, $V \approx 4.2097$ (c) Using 20 disks, $V \approx 4.1940$

6.2 Volumes by Cross-sections



Written Description	Draw and label the base. Draw the bottom of one of the slices.	Draw one slice and label its dimensions.	Write the integral for the volume.
The base is a circle of radius 2 centered about the origin. The cross sections perpendicular to the x-axis are squares.			$V=\int_a^b A_x\,dx$ $=\int_{-2}^{2} 4y^2\,dx$ $=\int_{-2}^{2} 4(4-x^2)\,dx$

6.2 Volumes by Cross-sections

EXAMPLE 7 **Triangular cross-sections** Figure 12 shows a solid with a circular base of radius 1. Parallel cross-sections perpendicular to the base are equilateral triangles. Find the volume of the solid.

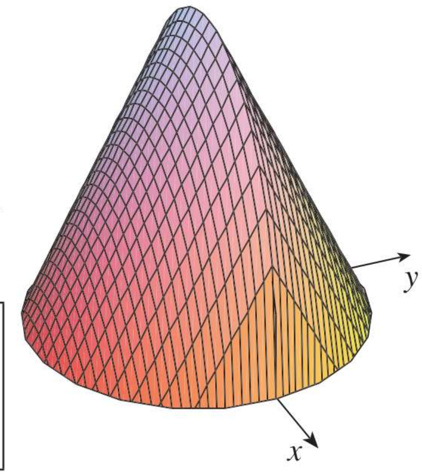
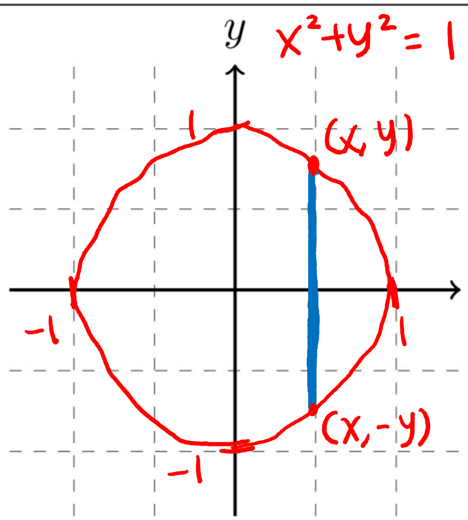
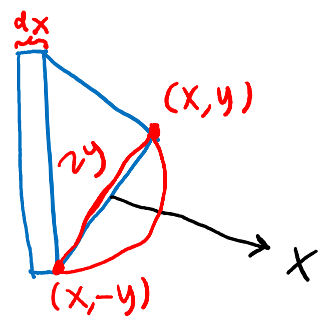


FIGURE 12

Draw and label the base. Draw the bottom of one of the slices.	Draw one slice and label its dimensions.	Write the integral for the volume.
	$A = \frac{1}{2}(2y)\, y\sqrt{3} = y^2\sqrt{3}$ 	$V = \int_{-1}^{1} A_x\,dx$ $= \int_{-1}^{1} y^2\sqrt{3}\,dx$ $= \int_{-1}^{1} (1-x^2)\sqrt{3}\,dx$

$$V = \sqrt{3}\int_{-1}^{1}(1-x^2)\,dx$$

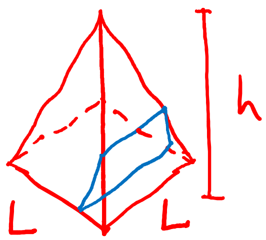
$$= \sqrt{3}\left[x - \frac{x^3}{3}\right]_{-1}^{1}$$

$$= \sqrt{3}\left[\left(1-\frac{1}{3}\right)-\left(-1+\frac{1}{3}\right)\right]$$

$$= \frac{4\sqrt{3}}{3}$$

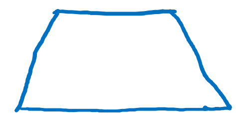
6.2 Volumes by Cross-sections

V EXAMPLE 8 Find the volume of a pyramid whose base is a square with side L and whose height is h.



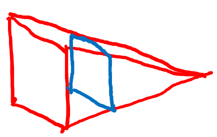
Warning

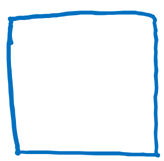
neither vertical nor horizontal slices in upright position are triangles.



parallelogram

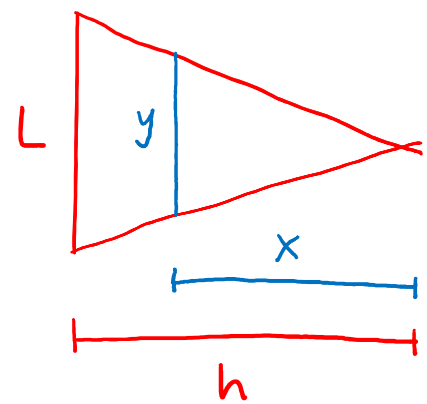
Observation: When the pyramid is lying down, the slices are now squares!





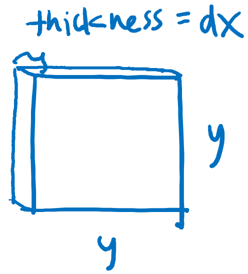
square

From the side, the pyramid looks like a triangle.



we can compute the length of the blue line by using similar triangles.

$$\frac{L}{h} = \frac{y}{x}$$



area $= y^2$

volume of slice $= y^2\,dx$

We sum the squares as we move along horizontally from 0 to h.

$$V = \int_0^h y^2\,dx = \int_0^h \left(\frac{Lx}{h}\right)^2 dx = \int_0^h \frac{L^2}{h^2} x^2\,dx = \frac{L^2}{h^2}\int_0^h x^2\,dx$$

$$= \frac{L^2}{h^2}\left[\frac{x^3}{3}\right]_0^h = \frac{L^2}{h^2}\left[\frac{h^3}{3} - 0\right] = \frac{1}{3}L^2 h$$

Class Activity

- Get into groups of 3-4
- You will receive a bag of 3D-printed objects. Handle them carefully, they can break apart easily.
- Have fun!