Daily Quiz

- Go to Socrative.com and complete the quiz.
- Room Name: HONG5824
- Use your full name.

6.1 Areas Review (Integrating along the x-axis)

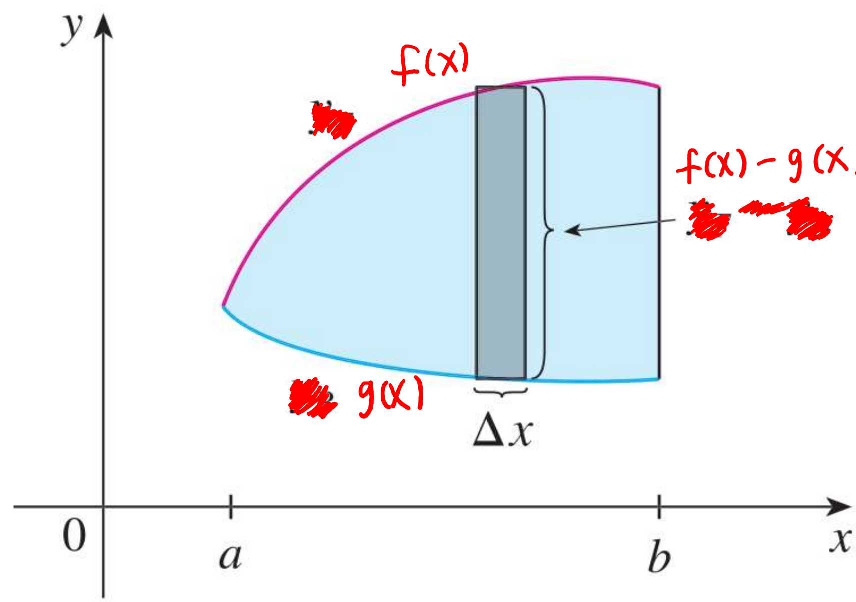
If $f(x) \geq g(x)$ for all x in the x-interval $[a, b]$, then the area between the two functions is

Top − Bottom

$$S = \int_a^b [f(x) - g(x)]dx$$

Since we are integrating along the x-axis, we say that $\Delta y = f(x) - g(x)$ is the height of a rectangle and dx is the width.

$$S = \int_a^b \Delta y \; dx$$



6.1 Areas Review (Integrating along the y-axis)

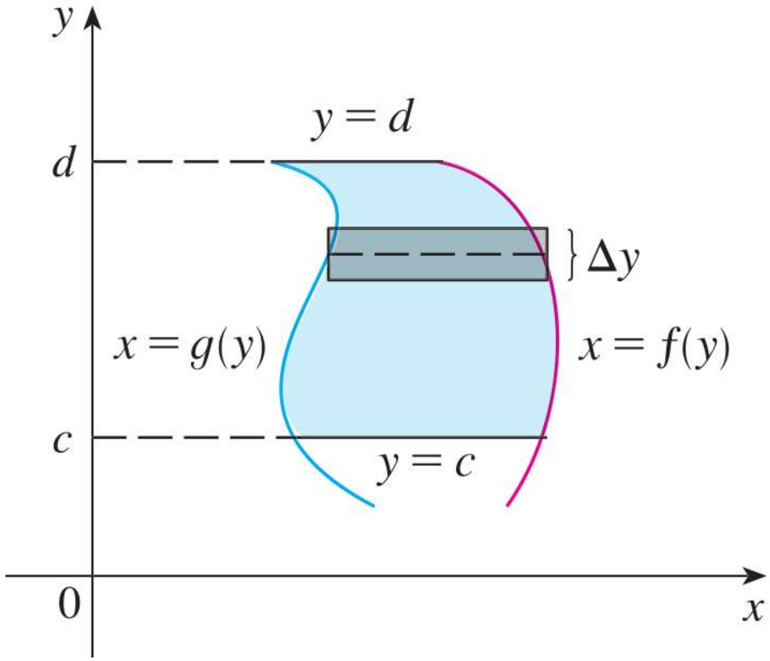
If $f(y) \geq g(y)$ for all y in the y-interval $[c, d]$, then the area between the two functions is

Right - Left

$$S = \int_c^d [f(y) - g(y)]dy$$

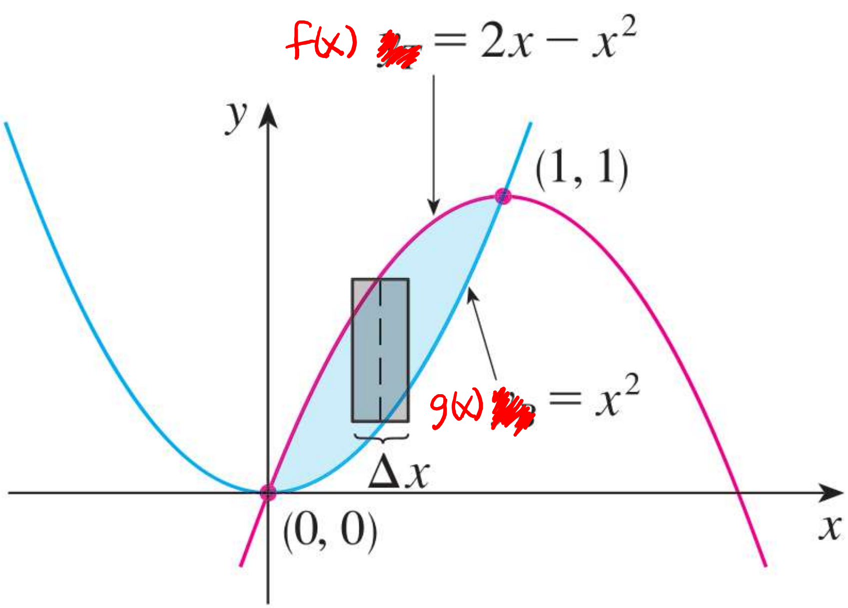
Since we are integrating along the y-axis, we say that $\Delta x = f(y) - g(y)$ is the width of a rectangle and dx is the height.

$$S = \int_c^d \Delta x \; dy$$



6.1 Areas Review

V EXAMPLE 2 Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.



① Find where they intersect (set the y-values equal)

$$x^2 = 2x - x^2$$
$$2x^2 - 2x = 0$$
$$2x(x-1) = 0$$

They intersect when $x = 0, 1$ so we integrate on $[0, 1]$.

② Integrate. Observe that $2x - x^2$ is above x^2 inside the interval $[0,1]$. Then

$$\text{Area} = \int_0^1 (2x - x^2) - (x^2) \, dx$$
$$= \int_0^1 2x - 2x^2 \, dx = \left[x^2 - \frac{2x^3}{3}\right]_0^1 = 1 - \frac{2}{3} = \frac{1}{3}.$$

6.1 Areas Review (Splitting into more than one integral)

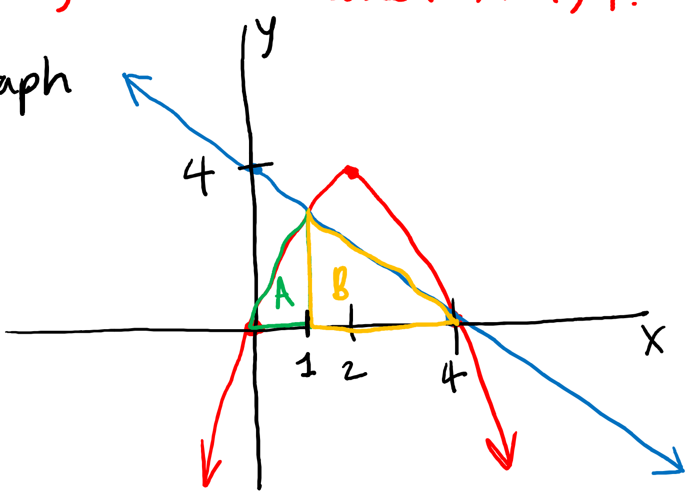
Find the area of the region bounded by $y = 4x - x^2$, $y = 4 - x$, and the x-axis.

① Find where they intersect (Set the y-values equal)

$$4x - x^2 = 4 - x$$
$$0 = x^2 - 5x + 4$$
$$0 = (x-4)(x-1)$$

They intersect when $x = 1, 4$.

Graph



② Integrate. Observe that the area A is bounded by the parabola and the x-axis while the area B is bounded only by the straight line and the x-axis. This means we need two separate integrals, one for each region.

$$\text{Area} = \int_0^1 \Big[\underbrace{(4x - x^2)}_{\text{parabola}} - \underbrace{0}_{\substack{y=0\\(x\text{-axis})}}\Big]\,dx + \int_1^4 \Big[\underbrace{(4-x)}_{\text{straight line}} - \underbrace{0}_{\substack{y=0\\(x\text{-axis})}}\Big]\,dx$$

$$= \left[2x^2 - \frac{x^3}{3}\right]_0^1 + \left[4x - \frac{x^2}{2}\right]_1^4$$

$$= \left(2 - \frac{1}{3}\right) - (0 - 0) + \left(16 - \frac{16}{2}\right) - \left(4 - \frac{1}{2}\right)$$

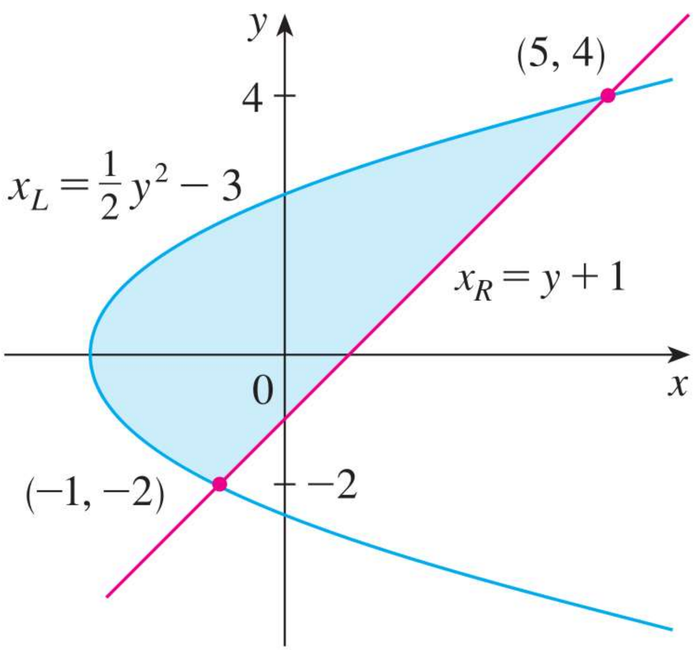
$$= \frac{5}{3} + \frac{16}{2} - \frac{7}{2}$$

$$= \frac{37}{6}$$

6.1 Areas Review

EXAMPLE 5 **Integrating with respect to *y* is sometimes easier**

Find the area enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.



① Find where they intersect.
(set the x-values equal)

$x = 1 + y$, $x = \frac{y^2-6}{2}$

$1 + y = \frac{y^2-6}{2}$

$2 + 2y = y^2 - 6$

$0 = y^2 - 2y - 8$

$0 = (y-4)(y+2)$

They intersect when
$y = -2, 4$.

Note that from the shape of the parabola, integrating along the y-axis looks easier.

The interval that we are integrating over is

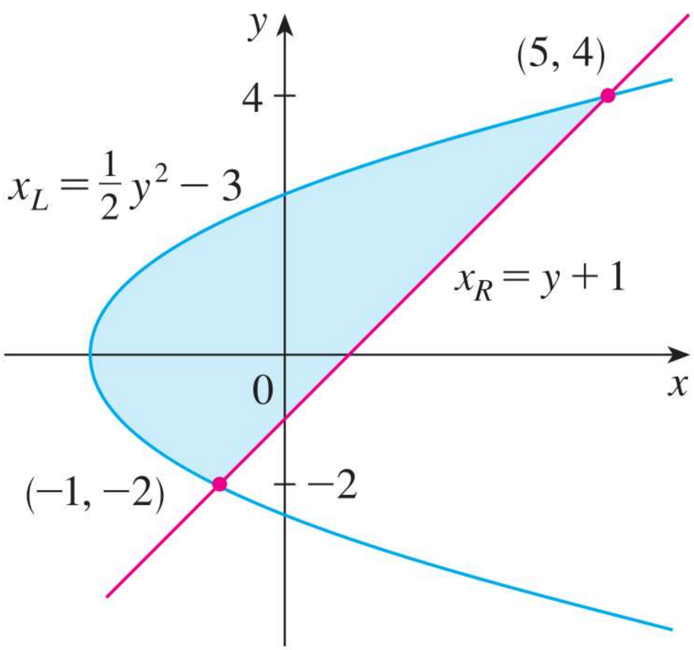
$[-2, 4]$ along

the y-axis.

6.1 Areas Review

EXAMPLE 5 **Integrating with respect to *y* is sometimes easier**

Find the area enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.



② Integrate. Observe that the line $y = x-1$ is to the right of the parabola $y^2 = 2x+6$

Then

$$\text{Area} = \int_{-2}^{4} \Delta x \, dy = \int_{-2}^{4} (y+1) - \left(\frac{y^2-6}{2}\right) dy$$

$$= \int_{-2}^{4} y - \frac{y^2}{2} + 4 \, dy = \left[\frac{y^2}{2} - \frac{y^3}{6} + 4y\right]_{-2}^{4}$$

$$= \left(\frac{16}{2} - \frac{64}{6} + 16\right) - \left(\frac{4}{2} + \frac{8}{6} - 8\right)$$

$$= 30 - \frac{72}{6}$$

$$= 18$$