Daily Quiz

- Go to Socrative.com and complete the quiz.
- Room Name: HONG5824
- Use your full name.

8.2 What can we do with sequences?

Let $\{a_n\}$ be a sequence. What can we do with sequences?

1. Infinite sums called **Series**

$$a_1 + a_2 + a_3 + \cdots = \sum_{n=1}^{\infty} a_n$$

2. Infinite products

$$a_1 \cdot a_2 \cdot a_3 \cdot \cdots = \prod_{n=1}^{\infty} a_n$$

8.2 Review of Sigma Notation

$$\sum_{k=1}^{n} a_k$$

8.2 Series

Definition. Given a sequence $\{a_k\}$, a finite sum

$$s_n = \sum_{k=1}^{n} a_k = a_1 + a_2 + \cdots + a_n$$

is called the n**-th partial sum** s_n.

Series is an infinite sum of the sequence $\{a_k\}$,

$$\sum_{k=1}^{\infty} a_k$$

8.2 Series

Since infinity is not a number, we define the series as the limit of the partial sums:

$$\sum_{k=1}^{\infty} a_k = \lim_{n\to\infty} \sum_{k=1}^{n} a_k$$

If the limit exists and is equal to a real number s, we say that the series **converges** and write

$$s = \sum_{k=1}^{\infty} a_k.$$

If the limit doesn't exist, then we say that the series is **divergent**.

8.2 Harmonic Series

V EXAMPLE 7 Show that the **harmonic series**

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$$

is divergent.

8.2 Harmonic Series

Another way to see the divergence of the **harmonic series** is to compare the expanded form to another series that diverges.

$$\begin{aligned}\sum_{n=1}^{\infty}\frac{1}{n} &= 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\cdots \\ &> 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{4}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{16}+\cdots \\ &= 1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\cdots \\ &= \infty\end{aligned}$$

Therefore $\sum_{n=1}^{\infty}\frac{1}{n} \geq \infty$ and the harmonic series **diverges**.

8.2 Telescoping Sum

Find the partial sum s_4 and show that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1.$

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8.2 The Divergence Test

Theorem. If the series $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.

The Divergence Test.

If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

(WRONG) If $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ is convergent.

Counterexample:

$$\lim_{n\to\infty} \frac{1}{n} = 0 \quad \text{but} \quad \sum_{n=1}^{\infty} \frac{1}{n} = \infty$$

8.2 The Divergence Test

Using the Test for Divergence Show that the series $\sum_{n=1}^{\infty} \frac{n^2}{5n^2 + 4}$ diverges.

8.2 The Divergence Test

Note 1: A finite number of terms doesn't affect the convergence or divergence of a series. What matters is the **long-term** behavior, not the short term.

Note 2: The converse of Theorem 6 is not true in general. If $\lim_{n\to\infty} a_n = 0$, we cannot conclude that $\Sigma\, a_n$ is convergent. Observe that for the harmonic series $\Sigma\, 1/n$ we have $a_n = 1/n \to 0$ as $n \to \infty$, but we showed in Example 7 that $\Sigma\, 1/n$ is divergent.

Note 3: If we find that $\lim_{n\to\infty} a_n \neq 0$, we know that $\Sigma\, a_n$ is divergent. If we find that $\lim_{n\to\infty} a_n = 0$, we know *nothing* about the convergence or divergence of $\Sigma\, a_n$. Remember the warning in Note 2: If $\lim_{n\to\infty} a_n = 0$, the series $\Sigma\, a_n$ might converge or it might diverge.

Useful Limits

1. $\lim_{x\to\infty} \sin x = \text{ DNE}$
2. $\lim_{x\to\infty} \arctan x = \dfrac{\pi}{2}$
3. $\lim_{x\to\infty} \left(1+\dfrac{1}{x}\right)^x = e$
4. $\lim_{x\to 0} \dfrac{\sin x}{x} = 1$
5. $\lim_{n\to\infty} \sqrt[n]{n} = 1$