Daily Quiz

- Go to Socrative.com and complete the quiz.
- Room Name: HONG5824
- Use your full name.

8.5 Power Series

Find the radius of convergence and the interval of convergence of the following series.

$$\sum_{n=1}^{\infty} \frac{(8x+3)^n}{n^2}$$

8.5 Power Series

Determine whether the statement is true or false.

If $\sum c_n 6^n$ is convergent, then $\sum c_n(-2)^n$ is convergent.

Find a power series representation for $f(x) = \dfrac{1}{1-x}$.

A function $f(x)$ is equal to its power series only for x in the interval of convergence.

Example: Let $f(x) = \dfrac{1}{1-x}$. Then

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

only for x in the interval of convergence $(-1, 1)$. If x is **not in the interval of convergence**, then the function is not equal to its power series at x:

$$\frac{1}{1-x} \neq \sum_{n=0}^{\infty} x^n$$

https://www.desmos.com/calculator/u0jgbq7jtj

8.6 Things you can do with power series

Substitution: Let $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$ and let u be a polynomial in x.

Then

$$f(u) = \sum_{n=0}^{\infty} c_n (u-a)^n$$

Express $\dfrac{1}{1+x^2}$ as a power series and find the interval of convergence.

Find a power series representation for $\dfrac{1}{x+2}$.

8.6 Things you can do with power series

Multiply by a polynomial in x: Let $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$ and let $p(x)$ be a polynomial in x. Then

$$p(x) \cdot f(x) = \sum_{n=0}^{\infty} p(x) \cdot c_n (x-a)^n$$

Note: Division is okay as long as there aren't any negative powers of x left over after simplification.

$$\frac{x^2 + x^3 + x^4 + \cdots}{x^2} = 1 + x + x^2 + \cdots \quad \text{POWER SERIES}$$

$$\frac{x^2 + x^3 + x^4 + \cdots}{x^3} = \frac{1}{x} + 1 + x + \cdots \quad \text{NOT A POWER SERIES}$$

Find a power series representation for $\dfrac{x^3}{x+2}$.

8.6 Things you can do with power series

Term-by-term Differentiation (Swapping the sum and the differential operator):

$$\frac{d}{dx}\left[\sum_{n=0}^{\infty} c_n(x-a)^n\right] = \sum_{n=0}^{\infty} \frac{d}{dx}[c_n(x-a)^n]$$

Express $\dfrac{1}{(1-x)^2}$ as a power series by differentiation. What is its interval of convergence?

Express $\dfrac{x}{(1+2x)^3}$ as a power series by differentiation.

8.6 Things you can do with power series

Term-by-term Integration (Swapping the sum and the integral):

$$\int \left[\sum_{n=0}^{\infty} c_n (x-a)^n \right] dx = \sum_{n=0}^{\infty} \int c_n (x-a)^n dx$$

Find a power series representation for $\ln(1 + x)$ and its interval of convergence.

Find a power series representation for $\arctan x$ and its interval of convergence.