Daily Quiz

- Go to Socrative.com and complete the quiz.
- Room Name: HONG5824
- Use your full name.

8.1 Sequences and Continuity

Continuity. If the function f is continuous at L and $\lim_{n\to\infty} a_n = L$, then

$$\lim_{n\to\infty} f(a_n) = f\left(\lim_{n\to\infty} a_n\right) = f(L).$$

In other words, limits can freely move in and out of continuous functions.

8.1 Sequences and Continuity

Find $\lim_{n \to \infty} \sin(\pi/n)$.

8.1 Factorials

The factorial of a positive integer n, denoted by $n!$, is the product of all **positive integers** less than or equal to n. If n is 0, then we define $0! = 1$. $n!$ is undefined if n is negative.

In other words,

$$n! = \begin{cases} n(n-1)\cdots 2\cdot 1 & \text{if } n \geq 1 \\ 1 & \text{if } n = 0 \\ \text{undefined} & \text{if } n < 0. \end{cases}$$

8.1 Sequences

Show that the sequence $a_n = \dfrac{n!}{n^n}$ converges.

8.1 Sequences

Show that the sequence $a_n = \dfrac{n!}{e^n}$ diverges.

8.1 Sequences

Definition A sequence $\{a_n\}$ is called **increasing** if $a_n < a_{n+1}$ for all $n \geqslant 1$, that is, $a_1 < a_2 < a_3 < \cdots$. It is called **decreasing** if $a_n > a_{n+1}$ for all $n \geqslant 1$. A sequence is **monotonic** if it is either increasing or decreasing.

8.1 Sequences

Show that the sequence $\left\{\dfrac{3}{n+5}\right\}_{n=1}^{\infty}$ is decreasing.

8.1 Sequences

Show that the sequence $a_n = \dfrac{n}{n^2+1}$ is decreasing.

8.1 Sequences

Definition A sequence $\{a_n\}$ is **bounded above** if there is a number M such that

$$a_n \leqslant M \qquad \text{for all } n \geqslant 1$$

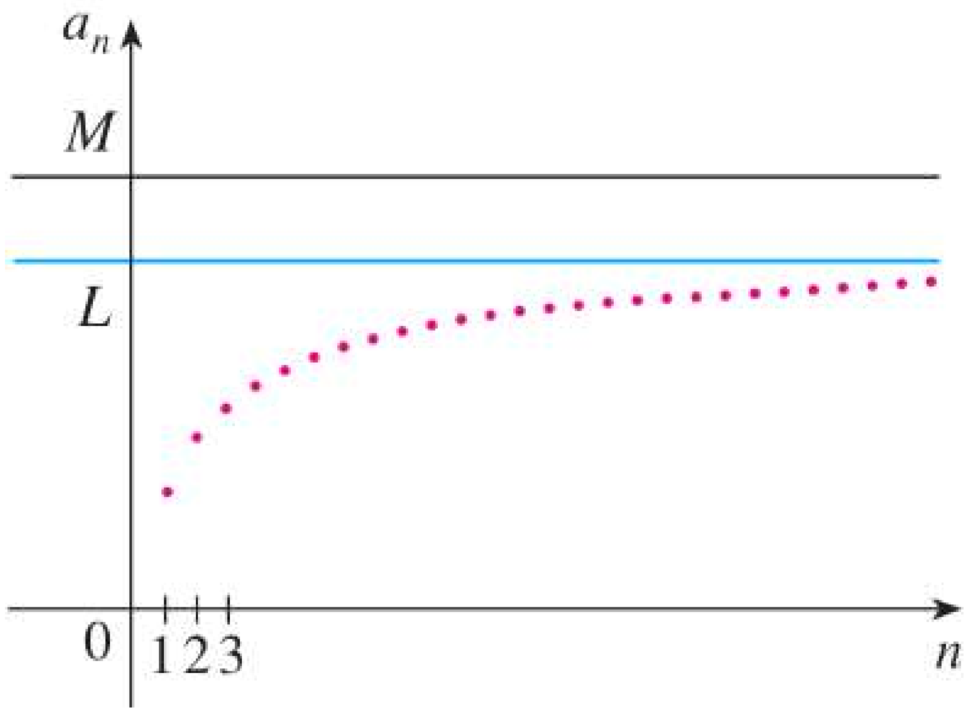
It is **bounded below** if there is a number m such that

$$m \leqslant a_n \qquad \text{for all } n \geqslant 1$$

If it is bounded above and below, then $\{a_n\}$ is a **bounded sequence**.

8.1 Sequences

8 **Monotonic Sequence Theorem** Every bounded, monotonic sequence is convergent.



8.1 Sequences

Find the limit of the sequence $\{a_n\}$ defined by the recurrence relation

$$a_1 = 2 \quad a_{n+1} = \frac{1}{2}(a_n + 6).$$

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