Daily Quiz

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- Use your full name.

8.4 Modes of Convergence

Absolutely Convergent

Definition. A series $\sum_{n=1}^{\infty} a_n$ is called **absolutely convergent** if the series of absolute values $\sum_{n=1}^{\infty} |a_n|$ is convergent.

Conditionally Convergent

Definition. A series $\sum_{n=1}^{\infty} a_n$ is called **conditionally convergent** if it is <u>not</u> absolutely convergent but still converges.

Divergent

Definition. A series $\sum_{n=1}^{\infty} a_n$ is divergent if the sequence of its partial sums $s_m = \sum_{n=1}^{m} a_n$ has no limit as n goes to infinity.

8.4 Absolute Convergence

$-1 \le \cos n \le 1$

$0 \le |\cos n| \le 1$

V EXAMPLE 7 Determine whether the series

$$\sum_{n=1}^{\infty} \frac{\cos n}{n^2} = \frac{\cos 1}{1^2} + \frac{\cos 2}{2^2} + \frac{\cos 3}{3^2} + \cdots$$

is convergent or divergent.

Test for absolute convergence.

abs $\displaystyle\sum_{n=1}^{\infty} \frac{|\cos n|}{n^2}$

Since $|\cos n| \ge 0$ for all n,

$\dfrac{|\cos n|}{n^2} \ge 0$ for all n.

Since $\dfrac{1}{n^2} \ge 0$ for all n and we have

$$\frac{|\cos n|}{n^2} \le \frac{1}{n^2},$$

we can apply the Direct Comparison Test

so $\displaystyle\sum_{n=1}^{\infty} \frac{|\cos n|}{n^2} \le \sum_{n=1}^{\infty} \frac{1}{n^2}$

and since $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges via p-test ($p=2$), the series $\sum_{n=1}^{\infty} \frac{|\cos n|}{n^2}$ converges.

Therefore $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$ converges absolutely.

Determine whether the series converges absolutely, converges conditionally, or diverges.

$$\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}}$$

We must first test for absolute convergence because the definition of conditional convergence requires a series to fail absolute convergence.

We have $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ as the absolute value version of the original series.

Now $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ is divergent via the p-test $(p=\frac{1}{2}<1)$ so we immediately see that $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}}$ is not absolutely convergent.

Since our series is an alternating series, let's see if it converges using the alternating series test.

Checking the hypotheses of the AST:

① vanishing at infinity

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$$

② Decreasing

$$b_n = \frac{1}{\sqrt{n}}, \quad f(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$$

$$f'(x) = -\frac{1}{2} \cdot x^{-\frac{3}{2}} = \frac{-1}{2x^{3/2}}$$

Since $f'(x) < 0$ for $x \geq 1$,

f is decreasing and so is

$b_n = \dfrac{1}{\sqrt{n}}$ for $n \geq 1$.

In conclusion, $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$ converges by the AST.

Since $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$ converges but is not absolutely convergent, it is conditionally convergent.