

# Exam 2 Review Session

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# Table of Contents

1. Average value of a function / Mean Value Theorem for Integrals
2. Work / Pressure / Center of Mass
3. Sequences and Series Overview
4. Series Matching Activity
5. Examples of absolute convergence, conditional convergence, and divergence.
6. Alternating Series Remainder Estimate
7. Integral Test Remainder Estimate

Given a function  $f(x)$ , the **average value of  $f$**  on the interval  $[a, b]$  is

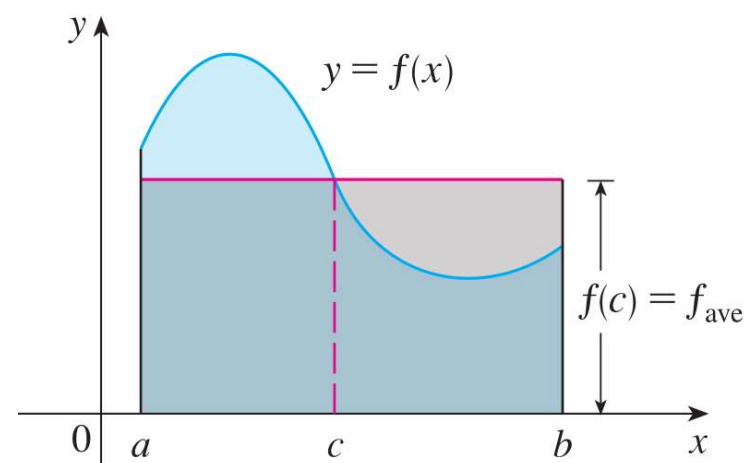
$$f_{\text{avg}} = \frac{1}{b - a} \int_a^b f(x) \, dx$$

**Mean Value Theorem for Integrals.** If  $f$  is continuous on  $[a, b]$ , then there exists a number  $c$  in  $[a, b]$  such that

$$f(c) = f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) dx$$

that is,

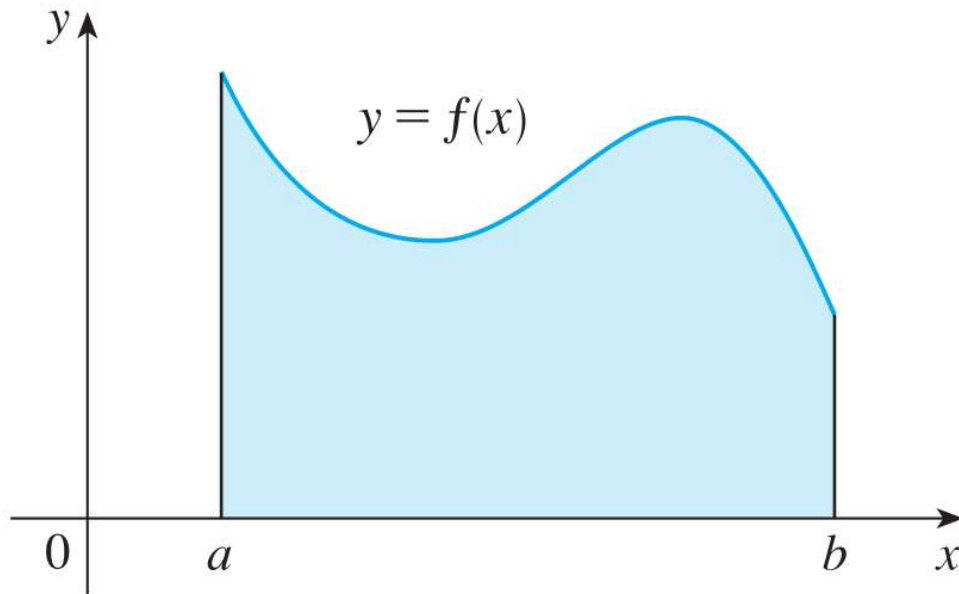
$$\int_a^b f(x) dx = f(c)(b-a)$$



**Geometric Interpretation.** For positive functions  $f$ , there is a number  $c$  such that the rectangle with base  $[a, b]$  and height  $f(c)$  has the same area as the region under the graph of  $f$  from  $a$  to  $b$ .

## 6.6 Work

- Work = Force x Distance.
- If force is a function that changes with respect to distance, then work can be thought of as the area under the curve.

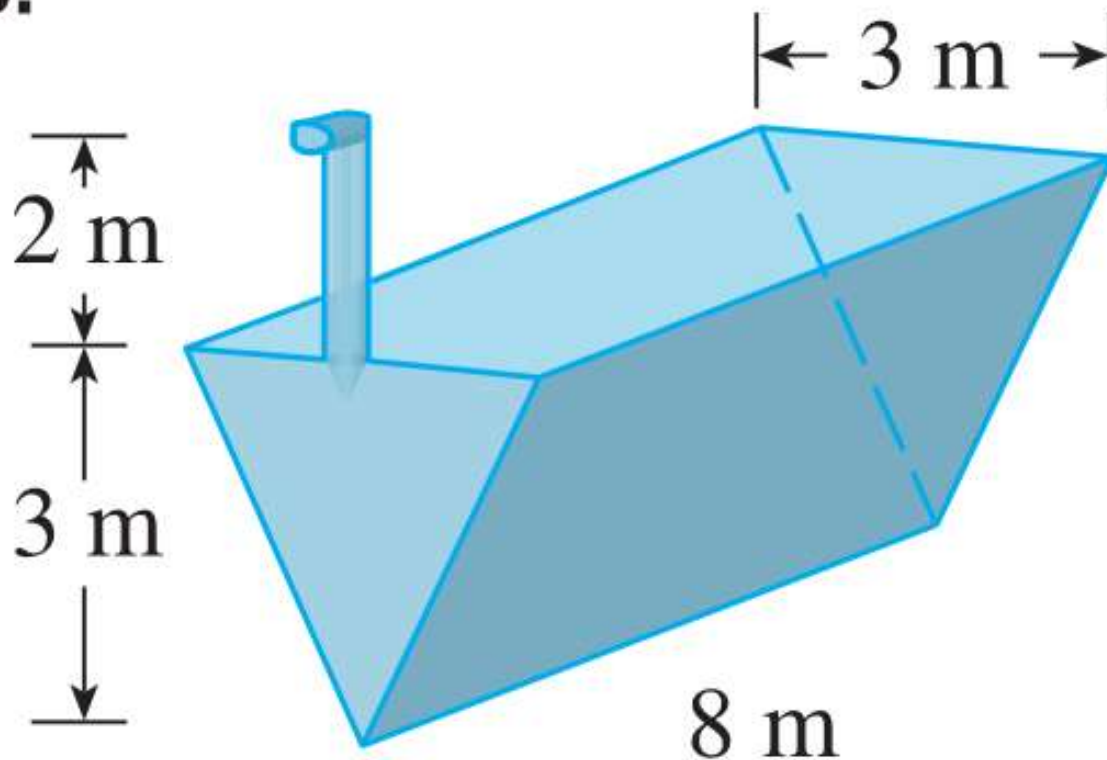


$$W = \int_a^b f(x) dx$$

## 6.6 Work

A tank is full of water. Find the work required to pump the water out of the spout.

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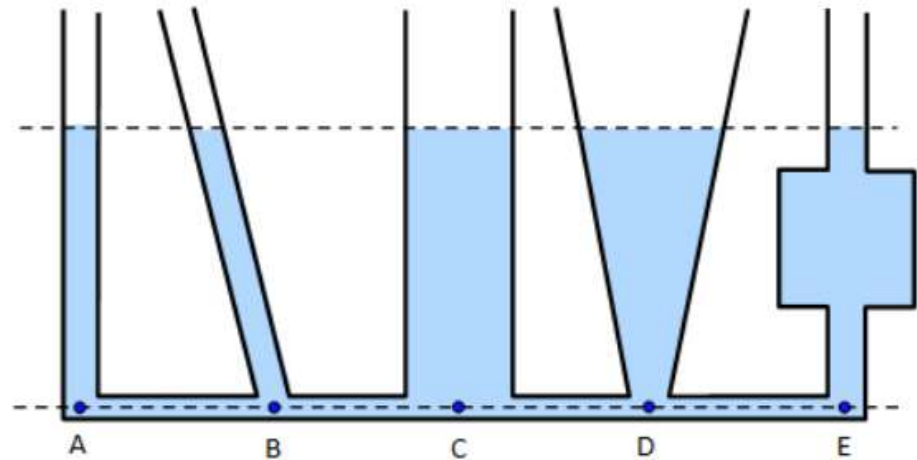




## 6.6 Hydrostatic Pressure and Force

At any point in a liquid the pressure is the same in ALL directions. The pressure of a liquid is the same at any given depth below the surface regardless of the shape of the container.

$$P = \frac{F}{A} = \rho g d$$

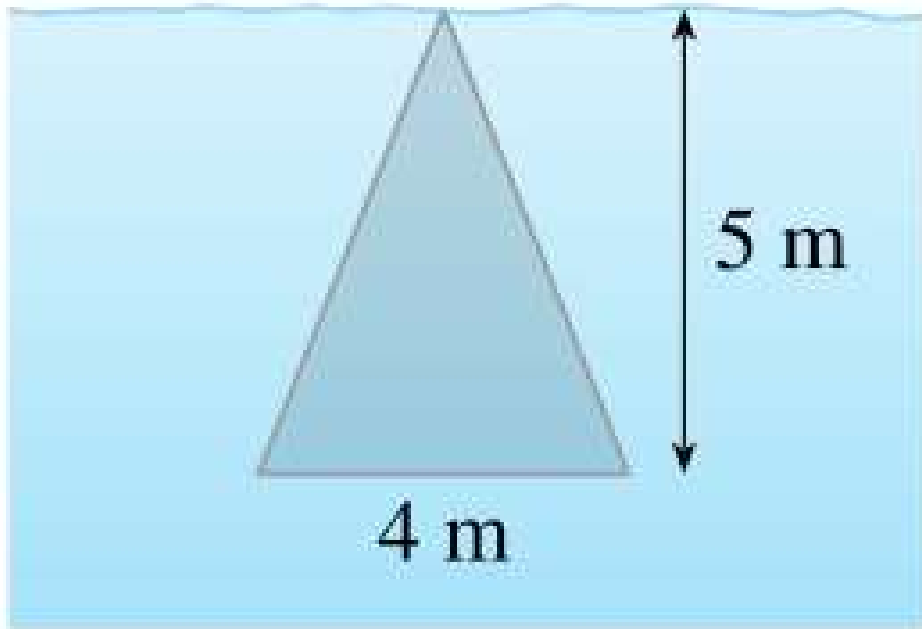


$\rho$  is the density of water,  $g$  is the gravitational constant, and  $d$  is the depth of the water.



## 6.6 Hydrostatic Pressure and Force

A triangle with base 4 m and height 5 m is submerged vertically in water so that the tip is even with the surface. Express the hydrostatic force against one side of the plate as an integral.





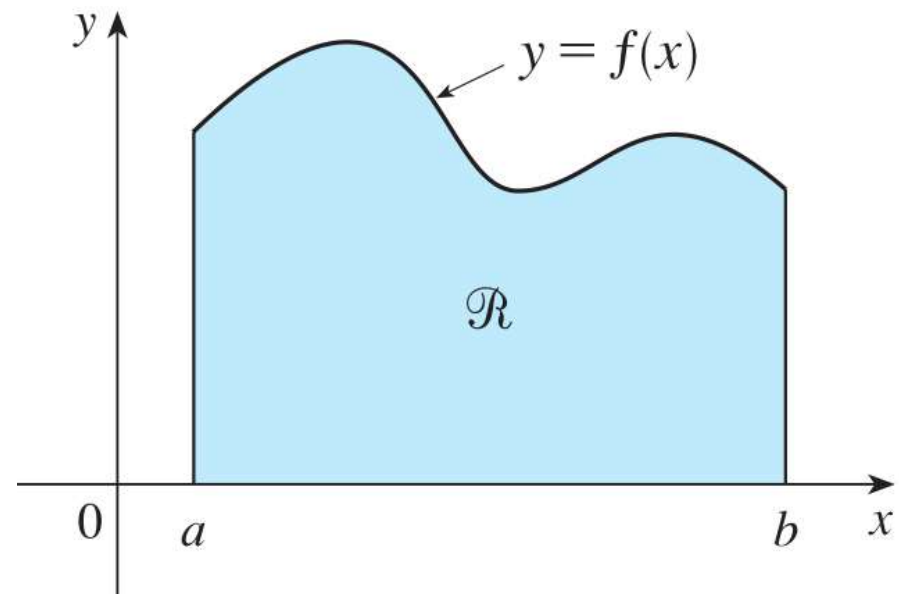
# Center of Mass

The center of mass of a region  $\mathcal{R}$  of constant density is located at  $(\bar{x}, \bar{y})$  and

$$\bar{x} = \frac{1}{A} \int_a^b x f(x) dx$$

$$\bar{y} = \frac{1}{A} \int_a^b \frac{1}{2} [f(x)]^2 dx$$

where  $A = \int_a^b f(x) dx$ .



# Computing the Center of Mass

Calculate the center of mass of the given lamina with constant density.

