Daily Quiz

- Go to Socrative.com and complete the quiz.
- Room Name: HONG5824
- Use your full name.

8.1 Sequences

A **sequence** is a list of numbers written in a definite order:

$$a_1, a_2, a_3, a_4, \cdots, a_n, \cdots$$

The elements a_n are called the **terms** of the sequence. Note that sequences don't end, and the terms $a_1, a_2, a_3, \cdots$ need not be distinct.

Given a sequence, it is customary to use $\{a_n\}$ instead of $a_1, a_2, a_3, \cdots$.

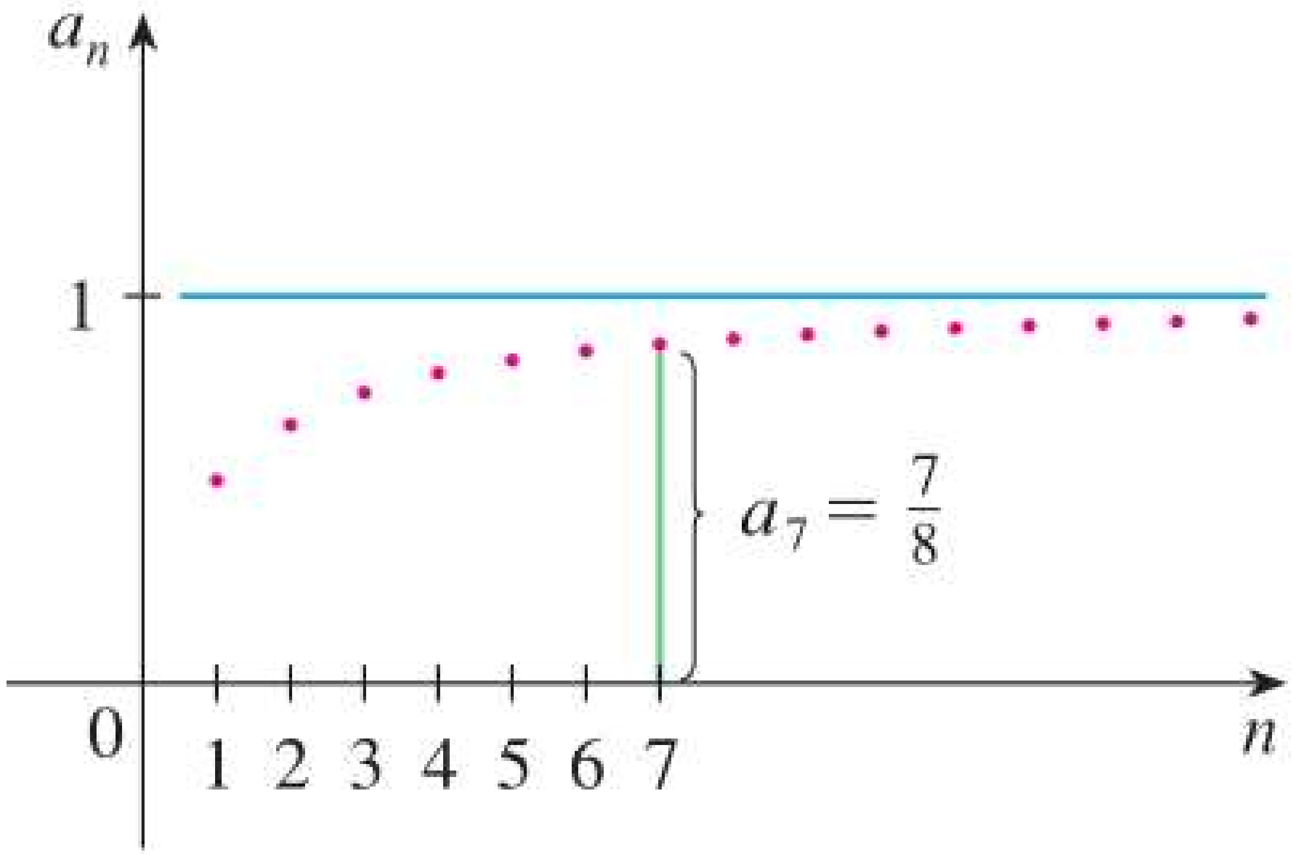
Examples:

Standard form	Expanded form	Formula
$\left\{\frac{n}{n+1}\right\}_{n=1}^{\infty}$	$\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots, \frac{n}{n+1}, \ldots\right\}$	$a_n = \frac{n}{n+1}$
$\left\{\frac{(-1)^n(n+1)}{3^n}\right\}_{n=1}^{\infty}$	$\left\{-\frac{2}{3}, \frac{3}{9}, -\frac{4}{27}, \frac{5}{81}, \ldots, \frac{(-1)^n(n+1)}{3^n}, \ldots\right\}$	$a_n = \frac{(-1)^n(n+1)}{3^n}$
$\left\{\sqrt{n-3}\right\}_{n=3}^{\infty}$	$\left\{0, 1, \sqrt{2}, \sqrt{3}, \ldots, \sqrt{n-3}, \ldots\right\}$	$a_n = \sqrt{n-3},\ n \geqslant 3$
$\left\{\cos\frac{n\pi}{6}\right\}_{n=0}^{\infty}$	$\left\{1, \frac{\sqrt{3}}{2}, \frac{1}{2}, 0, \ldots, \cos\frac{n\pi}{6}, \ldots\right\}$	$a_n = \cos\frac{n\pi}{6},\ n \geqslant 0$

8.1 Sequences

- What does a sequence look like?

$$a_n = \frac{n}{n+1}$$



8.1 Sequences

Find a formula for the general term a_n of the sequence

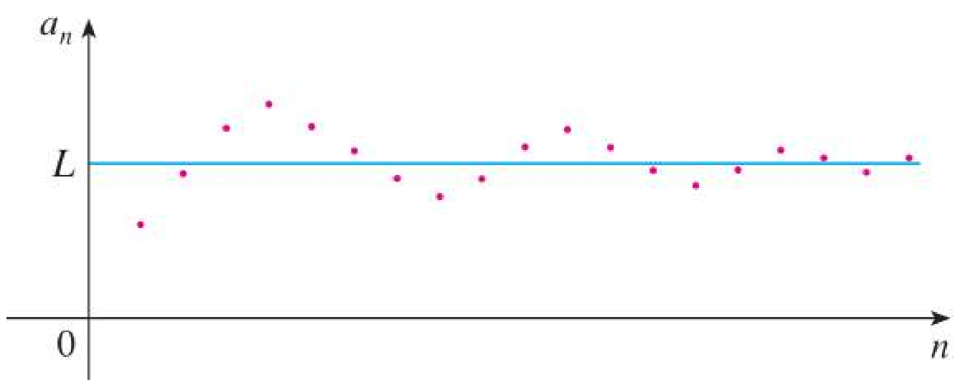
$$\left\{\frac{3}{5}, -\frac{4}{25}, \frac{5}{125}, -\frac{6}{625}, \frac{7}{3125}, \ldots\right\}$$

8.1 Sequences

1 Definition A sequence $\{a_n\}$ has the **limit** L and we write

$$\lim_{n \to \infty} a_n = L \qquad \text{or} \qquad a_n \to L \text{ as } n \to \infty$$

if we can make the terms a_n as close to L as we like by taking n sufficiently large. If $\lim_{n \to \infty} a_n$ exists, we say the sequence **converges** (or is **convergent**). Otherwise, we say the sequence **diverges** (or is **divergent**).



8.1 Sequences

Find $\lim_{n \to \infty} \dfrac{n}{n+1}$.

8.1 Sequences

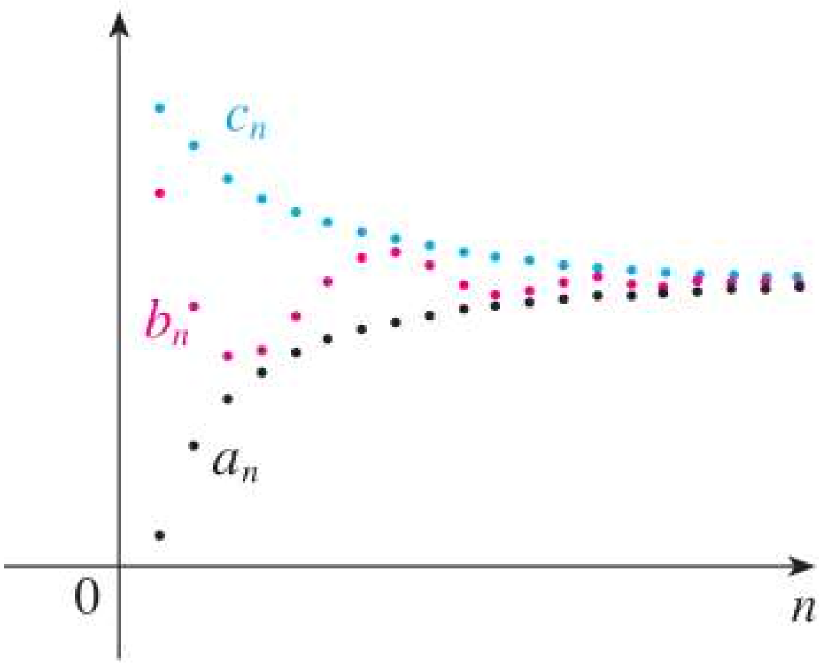
Applying l'Hospital's Rule to a related function Calculate $\lim_{n \to \infty} \frac{\ln n}{n}$.

8.1 Sequences

Determine whether the sequence $a_n = (-1)^n$ is convergent or divergent.

8.1 The Squeeze Theorem for Sequences

If $a_n \leqslant b_n \leqslant c_n$ for $n \geqslant n_0$ and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$, then $\lim_{n\to\infty} b_n = L$.



If $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$.

8.1 Sequences

Evaluate $\lim_{n\to\infty} \frac{(-1)^n}{n}$ if it exists.

8.1 Geometric Sequences

V EXAMPLE 10 **Limit of a geometric sequence** For what values of r is the sequence $\{r^n\}$ convergent?

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7 The sequence $\{r^n\}$ is convergent if $-1 < r \leqslant 1$ and divergent for all other values of r.

$$\lim_{n \to \infty} r^n = \begin{cases} 0 & \text{if } -1 < r < 1 \\ 1 & \text{if } r = 1 \end{cases}$$